CHAPTER 14

Trigonometry

LEARNING OBJECTIVES

After completing this chapter, you will be able to:

- Calculate trigonometric ratios from side lengths of right triangles
- Calculate side lengths of right triangles using trigonometric ratios
- Describe the relationship between the sine and cosine of complementary angles

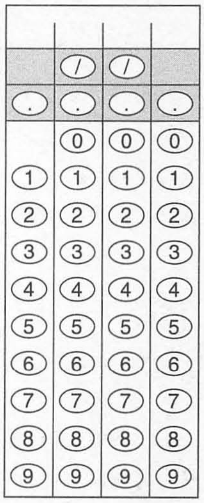
5/600 SmartPoints® (Low Yield)

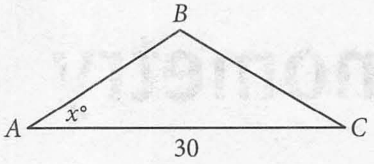
Math

How Much Do You Know?

Directions: Try the questions that follow. Show your work so that you can compare your solutions to the ones found in the Check Your Work section immediately after this question set. If you answered most of the questions correctly, and if your scratchwork looks like ours, you may be able to move quickly through this chapter. If you answered incorrectly or used a different approach, you may want to take your time on this chapter.

1. If $\tan x = \frac{7}{24}$, what is the value of $\sin x$?





Note: Figure not drawn to scale.

2. If the area of $\triangle ABC$ is 225 and $AB = 17$, what is the value of $\cos x$?

A) $\frac{8}{17}$

B) $\frac{8}{15}$

C) $\frac{17}{30}$

D) $\frac{15}{17}$

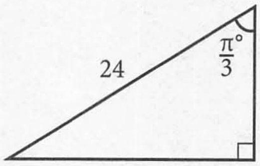
3. In a right triangle, one of the acute angles is $\cos\left(\frac{\pi}{3}\right)$, and $\cos\left(\frac{\pi}{3}\right) = \sin x$. What is the measure of x?

A) $\frac{\pi}{12}$

B) $\frac{\pi}{6}$

C) $\frac{\pi}{3}$

D) $\frac{2\pi}{3}$



4. If the hypotenuse of the triangle shown above has length 24 units, what is the area in square units of the triangle?

A) $72\sqrt{3}$

B) $144\sqrt{3}$

C) 288

D) $288\sqrt{3}$

Answers and explanations are on the next page. ▶ ▶ ▶

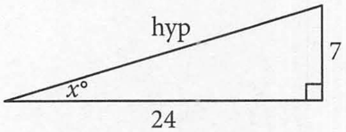
Math

Check Your Work

1. 7/25 or .28

Difficulty: Medium

Getting to the Answer: Recall that $\tan x = \frac{\text{opp}}{\text{adj}}$. This means you know the lengths of the two legs of the triangle, but not its hypotenuse. Draw a picture with the given lengths:



Remember that $\sin x = \frac{\text{opp}}{\text{hyp}}$, so use the Pythagorean theorem to calculate the length of the hypotenuse:

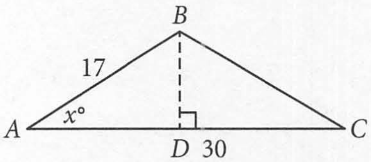
$$7^2 + 24^2 = \text{hyp}^2$$
$$49 + 576 = \text{hyp}^2$$
$$\sqrt{625} = \sqrt{\text{hyp}^2}$$
$$25 = \text{hyp}$$

Plug the length of the hypotenuse into the sine ratio: $\sin x = \frac{7}{25}$. Grid in **7/25** or **.28**.

2. A

Difficulty: Hard

Getting to the Answer: Begin by drawing a height in the triangle:



Next, find the length of *BD* using the triangle area formula:

$$A = \frac{1}{2}bh$$
$$225 = \frac{1}{2} \times 30h$$
$$h = \frac{225}{15} = 15 = BD$$

You now have two sides of a right triangle ($\triangle ABD$)—but not necessarily the two sides you need. To find the value of cos *x*, you need the side adjacent to *x* and the hypotenuse. Here, you have the opposite side (*BD*) and the hypotenuse (*AB*), so you need to find the length of the third side (*AD*). You might recognize $\triangle ABD$ as an 8-15-17 Pythagorean triple, but if you don't, you can use the Pythagorean theorem:

$$AD^2 + 15^2 = 17^2$$
$$AD^2 = 17^2 - 15^2$$
$$AD^2 = 289 - 225$$
$$AD = \sqrt{64} = 8$$

You can now find cos *x*: $\frac{\text{adj}}{\text{hyp}} = \frac{8}{17}$. **(A)** is correct.

3. B

Difficulty: Medium

Getting to the Answer: The sine and cosine of complementary angles are equal, and the sum of the acute angles in a right triangle is $180° - 90° = 90°$. Convert this to radians to find the measure of the missing angle. The sum of the acute angles in a right triangle, in radians, is $90° \times \frac{\pi}{180°} = \frac{90\pi}{180} = \frac{\pi}{2}$. Subtract the known angle to find the other angle: $\frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi}{6} - \frac{2\pi}{6} = \frac{\pi}{6}$. **(B)** is correct.

4. A

Difficulty: Medium

Getting to the Answer: When the measure of an angle in a triangle is given in radians, you'll usually want to convert it to degrees because you might be able to find a special right triangle. Use the relationship $180° = \pi$ to convert the angle: $\frac{\not{\pi}}{3} \times \frac{180°}{\not{\pi}} = 60°$.

Now you know the triangle is a 30-60-90 triangle, which has sides that are in the ratio $x : x\sqrt{3} : 2x$. The hypotenuse is $2x = 24$, so $x = 12$ and $x\sqrt{3} = 12\sqrt{3}$. Therefore, the base and height of the triangle are 12 and $12\sqrt{3}$, and the area of the triangle is $\frac{1}{2}(12\sqrt{3})(12) = 72\sqrt{3}$, so **(A)** is correct. Note that because it is a right triangle, it does not matter which leg you call the base and which the height.

Sine, Cosine, and Tangent

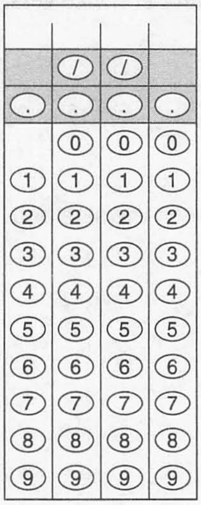
LEARNING OBJECTIVES

After this lesson, you will be able to:

- Calculate trigonometric ratios from side lengths of right triangles
- Calculate side lengths of right triangles using trigonometric ratios
- Describe the relationship between the sine and cosine of complementary angles

To answer a question like this:

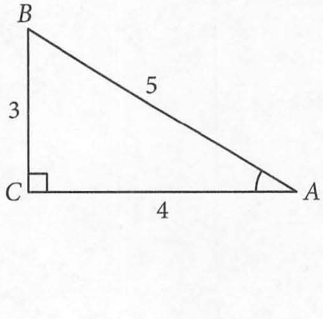
One angle in a right triangle measures $y°$ such that $\cos y° = \frac{24}{25}$. What is the measure of $\sin(90° - y°)$?



You need to know this:

The SAT tests three trigonometric functions: **sine, cosine**, and **tangent**. All three are simply the ratios of side lengths within a right triangle. The notation for sine, cosine, and tangent functions always includes a reference angle; for example, $\cos x$ or $\cos\theta$. That's because you'll need to refer to the given angle within a right triangle to determine the appropriate side ratios.

There is a common mnemonic device for the sine, cosine, and tangent ratios: SOHCAHTOA (commonly pronounced: so-kuh-TOE-uh). Here's what it represents: **S**ine is **O**pposite over **H**ypotenuse, **C**osine is **A**djacent over **H**ypotenuse, and **T**angent is **O**pposite over **A**djacent. See the following triangle and the table for a summary of the ratios and what each equals for angle *A* in triangle *CAB*:



Sine (sin)	Cosine (cos)	Tangent (tan)
$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$	$\frac{\text{opposite}}{\text{adjacent}}$
$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{4}$

Complementary angles have a special relationship relative to sine and cosine:

- $\sin x^\circ = \cos(90^\circ - x^\circ)$
- $\cos x^\circ = \sin(90^\circ - x^\circ)$

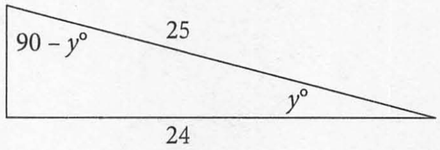
In other words, the sine of an acute angle is equal to the cosine of the angle's complement and vice versa. For example, $\cos 30^\circ = \sin 60^\circ$, $\cos 45^\circ = \sin 45^\circ$, and $\cos 60^\circ = \sin 30^\circ$.

You need to do this:

Apply the appropriate trigonometric ratio to a right triangle or use the relationship between the sine and cosine of complementary angles.

Explanation:

There are two ways to approach this question. You might choose to draw the triangle:



To find $\sin(90^\circ - y^\circ)$, put the side opposite the angle labeled $90^\circ - y^\circ$ over the hypotenuse. You'll get $\frac{24}{25}$, exactly the same as $\cos y^\circ$.

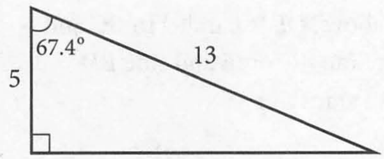
Alternatively, you could use the property of complementary angles that says that $\cos x^\circ = \sin(90^\circ - x^\circ)$ to find that $\sin(90^\circ - y^\circ) = \frac{24}{25}$.

The fraction can't be gridded in because it takes too many spaces, so divide 24 by 25 and grid in the result, **.96**. Although it doesn't apply here, pay attention to any rounding guidelines in the question stem.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: Pythagorean triples frequently appear in trig questions. What is the triangle's missing side length in Q1?



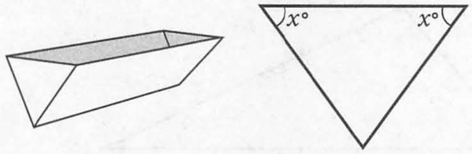
1. Based on the figure above, which of the following is true?

A) $\sin 22.6° = \frac{5}{12}$

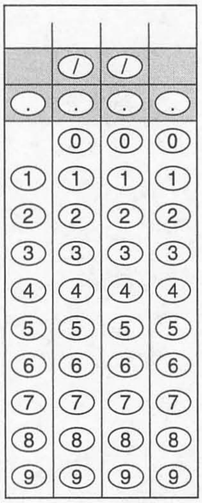
B) $\sin 67.4° = \frac{5}{13}$

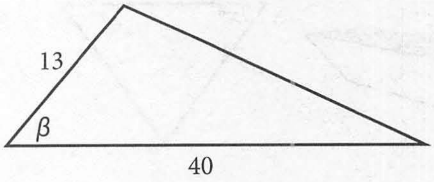
C) $\cos 22.6° = \frac{5}{13}$

D) $\cos 67.4° = \frac{5}{13}$



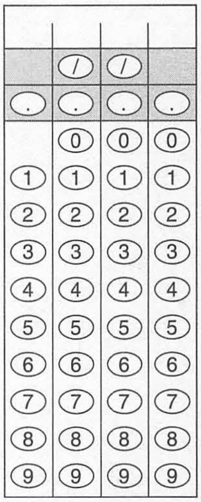
2. The triangle shown above is a cross section of a feeding trough. The triangular cross section is 24 inches deep and 36 inches across the top. If $\cos x = B$, what is the value of B?



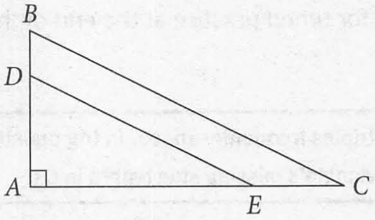


Note: Figure not drawn to scale.

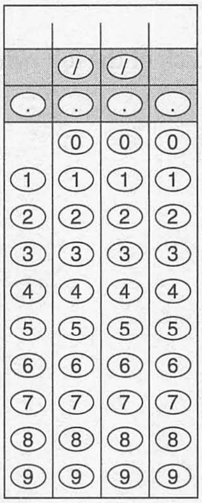
3. If the area of the triangle shown above is 240 square inches, what is $\tan \beta$?

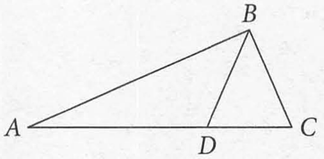


HINT: For Q4, what do you know about triangles with a shared angle and parallel sides?



4. In the figure above, DE is parallel to BC and $\sin \angle C = 0.6$. Side $AC = 16$ and side $BD = 3$. What is the length of side AE?





5. If $\sin \angle A = \cos \angle C$, what is $\sin \angle ABD - \cos \angle DBC$?

A) 0

B) $\frac{1}{2}$

C) 1

D) The result of the subtraction cannot be determined without additional information.

On Test Day

Occasionally, a question will give you more information than you need to determine the correct answer. Think about what information you really need to arrive at the answer before you begin your calculations so that you don't get sidetracked and spend time doing unnecessary work.

As you read through this question, plan your strategy to get the correct value and identify what information you need to carry out that strategy. Note if there is any unnecessary information that you can ignore.

6. Triangle *PQR* is a right triangle with the 90° angle at vertex *Q*. The length of side PQ is 25 and the length of side QR is 60. Triangle *STU* is similar to triangle *PRQ*. The vertices *S*, *T*, and *U* correspond to vertices *P*, *Q*, and *R*, respectively. Each side of triangle *STU* is $\frac{1}{10}$ the length of the corresponding side of triangle *PRQ*. What is the value of $\cos \angle U$?

A) $\frac{5}{13}$

B) $\frac{5}{12}$

C) $\frac{5}{6}$

D) $\frac{12}{13}$

The answer and explanation can be found at the end of this chapter.

How Much Have You Learned?

Directions: For testlike practice, give yourself 7 minutes to complete this question set. Be sure to study the explanations, even for questions you got right. They can be found at the end of this chapter.

7. If $\sin x = \cos\left(\frac{13\pi}{6}\right)$, which of the following could be the value of x?

A) $\frac{\pi}{6}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{3}$

D) $\frac{\pi}{2}$

8. If $\cos x = \sin y$, then which of the following pairs of angle measures could NOT be the values of x and y, respectively?

A) $\frac{\pi}{4}, \frac{\pi}{4}$

B) $\frac{\pi}{6}, \frac{\pi}{3}$

C) $\frac{\pi}{8}, \frac{3\pi}{8}$

D) $\frac{\pi}{2}, \frac{\pi}{2}$

9. Angle x is one of the acute angles in a right triangle. If the measure of angle x is 30°, what is the value of $(\sin x)^2 + (\cos x)^2$?

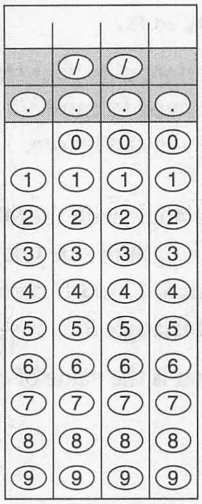
A) $\frac{1}{4}$

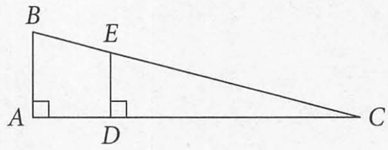
B) $\frac{1}{2}$

C) 1

D) 2

10. In a certain triangle, the measures of $\angle A$ and $\angle B$ are $(6k - 8)°$ and $(7k - 45)°$, respectively. If $\frac{\sin \angle A}{\cos \angle B} = 1$, what is the value of k?





11. In the above triangle, if side $AB = 5$, segment $AD = 3$, and $\tan \angle B = 2.4$, what is the length of segment BE?

A) 3

B) $3\frac{1}{4}$

C) $3\frac{3}{4}$

D) $4\frac{1}{4}$

Reflect

Directions: Take a few minutes to recall what you've learned and what you've been practicing in this chapter. Consider the following questions, jot down your best answer for each one, and then compare your reflections to the expert responses on the following page. Use your level of confidence to determine what to do next.

What are the definitions of sine, cosine, and tangent?

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____

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What is the special relationship of sine to cosine in complementary angles?

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____

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Expert Responses

What are the definitions of sine, cosine, and tangent?

Sine is defined as opposite over hypotenuse, cosine as adjacent over hypotenuse, and tangent as opposite over adjacent. The acronym SOHCAHTOA can help you remember these definitions.

What is the special relationship of sine to cosine in complementary angles?

For two complementary angles, $\sin x° = \cos(90° - x°)$ *and* $\cos x° = \sin(90° - x°)$. *In other words, if two angles are complementary (add up to 90 degrees), the sine of one equals the cosine of the other.*

Next Steps

If you answered most questions correctly in the "How Much Have You Learned?" section, and if your responses to the Reflect questions were similar to those of the SAT expert, then consider trigonometry an area of strength and move on to the next chapter. Come back to this topic in a few days to prevent yourself from getting rusty.

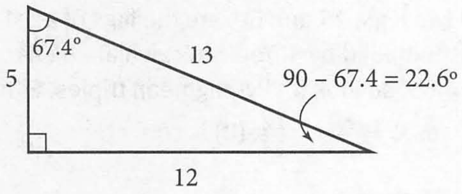
If you don't yet feel confident, review those parts of this chapter that you have not yet mastered, then try the questions you missed again. As always, be sure to review the explanations closely.

Answers and Explanations

1. D

Difficulty: Medium

Getting to the Answer: Find the unknown leg length and angle measure. The triangle is a right triangle with one leg length of 5 and a hypotenuse of 13, so the other leg is length 12. (If you didn't see the Pythagorean triple 5:12:13, you could have used the Pythagorean theorem to find the missing leg length.) Use the measures of the internal angles to find the missing angle:

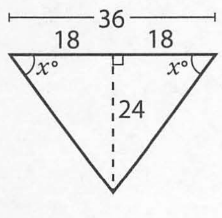


Sine and cosine both involve the hypotenuse, 13, so you can eliminate (A). Compare the remaining answer choices to the trig ratios given by SOHCAHTOA. Sine is opposite over hypotenuse, but the side opposite the 67.4° angle has length 12 (not 5), so eliminate (B). Cosine is adjacent over hypotenuse, but the side adjacent to the 22.6° angle has length 12 (not 5), so eliminate (C). Only **(D)** is left and must be correct. For the record, the side adjacent to the 67.4° angle has length 5 and the hypotenuse has length 13, so $\cos 67.4° = \frac{5}{13}$.

2. 3/5 or .6

Difficulty: Hard

Getting to the Answer: Because trig functions typically apply to right triangles, draw in an altitude and label what you know. You know the trough is 24 inches deep and 36 inches across the top. Because the given angles have equal measures, $x°$, the triangle is isosceles and the altitude bisects the top. Draw a figure:



You're given that $B = \cos x$, and the cosine of an angle involves the hypotenuse, so you need to find the length of the hypotenuse using the Pythagorean theorem:

$$18^2 + 24^2 = c^2$$
$$324 + 576 = c^2$$
$$\sqrt{900} = \sqrt{c^2}$$
$$30 = c$$

Finally, $\cos x = \frac{\text{adj}}{\text{hyp}} = \frac{18}{30} = \frac{3}{5}$. Grid in **3/5** or **.6**.

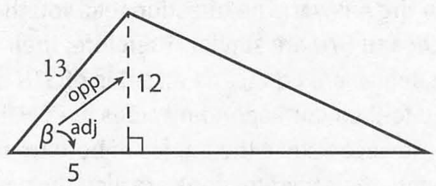
3. 12/5 or 2.4

Difficulty: Hard

Getting to the Answer: Find the height of the triangle using the information given about the area and add it to the figure:

$$A = \frac{1}{2}bh$$
$$240 = \frac{1}{2}(40)h$$
$$240 = 20h$$
$$12 = h$$

After you find the height, you might recognize the 5-12-13 Pythagorean triple, which gives you another side of the triangle that contains β:



Note: Figure not drawn to scale.

Now use SOHCAHTOA: $\tan \beta = \frac{\text{opp}}{\text{adj}} = \frac{12}{5}$. Grid in **12/5** or **2.4**.

Math

4. 12

Difficulty: Hard

Getting to the Answer: The fact that *DE* is parallel to *BC* means that triangles *ABC* and *ADE* are similar. Convert sin *C*, 0.6, to a fraction, $\frac{3}{5}$. Because sin is $\frac{\text{opposite}}{\text{hypotenuse}}$, both triangles have the side ratio 3:4:5. The question states that $AC = 16$. This is the long leg of a 3:4:5 right triangle, so $AB = 12$ and $BC = 20$.

The other known dimension is $BD = 3$. Since the length of *AB* is 12, the length of *AD* is $12 - 3 = 9$. Thus, the ratio of the sides of triangle *ADE* to those of triangle *ABC* is $\frac{9}{12} = \frac{3}{4}$. Therefore, *AD* is $\frac{3}{4}$ of *AC*, which is $\frac{3}{4} \times 16 = 12$. Grid in **12**.

5. A

Difficulty: Hard

Getting to the Answer: The sine of an angle is equal to the cosine of its complementary angle, so $\angle A + \angle C = 90°$. Since $\angle B$ is the third interior angle of the triangle *ABC*, $\angle B = 180° - 90° = 90°$. Therefore, the measures of angles *ABD* and *DBC* must total 90°, which means they are complementary angles. Thus, $\sin \angle ABD = \cos \angle DBC$, and $\sin \angle ABD - \cos \angle DBC = 0$. **(A)** is correct.

6. D

Difficulty: Medium

Getting to the Answer: The question tells you that triangles *PQR* and *STU* are similar. Therefore, their corresponding angles are equal and all sides of *STU* have the same ratio to their corresponding sides in *PQR*, in this case, $\frac{1}{10}$. However, since the angles of both triangles are the same, their trig functions are also the same, so there is no need to calculate the lengths of the sides of triangle *PQR*. Because they are the same ratio, cosine of *U* will have the same value as the cosine of *R*, so just calculate the cosine of *R* value using the side lengths of triangle *PQR*.

Since $\angle Q = 90°$, sides *PQ* and *QR* are legs of the right triangle and *PR* is the hypotenuse. (If you have trouble visualizing this, you can draw a very quick sketch.) Thus, $\cos R = \frac{QR}{PR}$. You can calculate the value of *PR* using the Pythagorean theorem:

$$\begin{aligned} 25^2 + 60^2 &= PR^2 \\ 625 + 3{,}600 &= PR^2 \\ \sqrt{4{,}225} &= \sqrt{PR^2} \\ PR &= 65 \end{aligned}$$

You can save a lot of time calculating if you recognize that the two legs, 25 and 60, are the legs of a 5:12:13 triangle multiplied by 5. You can calculate *PR* as $5 \times 13 = 65$, so look for Pythagorean triples. Either way, $\cos U = \cos R = \frac{60}{65} = \frac{12}{13}$. **(D)** is correct.

7. C

Difficulty: Hard

Getting to the Answer: Dealing with smaller angles usually makes trig questions easier, so start by subtracting 2π from $\frac{13\pi}{6}$ to get $\frac{13\pi}{6} - 2\pi = \frac{13\pi}{6} - \frac{12\pi}{6} = \frac{\pi}{6}$. (This is permissible because 2π is once around the unit circle; you are not changing the quadrant of the angle.) The equation in the question stem becomes $\sin x = \cos\left(\frac{\pi}{6}\right)$. Complementary angles have a special relationship relative to trig values—the cosine of an acute angle is equal to the sine of the angle's complement and vice versa. The angle measures are given in radians, so you're looking for an angle that, when added to $\frac{\pi}{6}$, gives $\frac{\pi}{2}$ (because $\frac{\pi}{2} = 90°$). Because $\frac{\pi}{6} + \frac{2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$, the two angles, $\frac{\pi}{6}$ and $\frac{\pi}{3}$, are complementary angles, which means **(C)** is correct.

8. D

Difficulty: Medium

Getting to the Answer: Use the special relationship that complementary angles have in terms of trig functions: the cosine of an acute angle is equal to the sine of the angle's complement and vice versa.

The question asks for the pair of angles that are *not* complementary. In degrees, complementary angles add up to 90, so in radians they add up to $90° \times \frac{\pi}{180°} = \frac{\pi}{2}$. The only pair of angles that is not complementary is **(D)**, the correct answer, because they add up to $\frac{\pi}{2} + \frac{\pi}{2} = \frac{2\pi}{2} = \pi$.

9. C

Difficulty: Medium

Getting to the Answer: Use the definitions of the common trig functions to put some context to this question. Substitute $\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)^2 + \left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)^2$ for $(\sin x)^2 + (\cos x)^2$. This simplifies to $\frac{\text{opposite}^2 + \text{adjacent}^2}{\text{hypotenuse}^2}$. The Pythagorean theorem states $a^2 + b^2 = c^2$, which is equivalent to $\text{opposite}^2 + \text{adjacent}^2 = \text{hypotenuse}^2$, so this fraction is actually $\frac{\text{hypotenuse}^2}{\text{hypotenuse}^2} = 1$. **(C)** is correct.

If you prefer to work with actual values, recall that the side ratio of a 30-60-90 triangle is $1:\sqrt{3}:2$. So, $\sin x = \frac{1}{2}$ and $\cos x = \frac{\sqrt{3}}{2}$. The question asks for the sum of the squares of those values: $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$. **(C)** is indeed correct.

10. 11

Difficulty: Hard

Getting to the Answer: If $\frac{\sin A}{\cos B} = 1$, then $\sin A = \cos B$, so $\angle A$ and $\angle B$ are complementary. Therefore, the sum of the measures of $\angle A$ and $\angle B$ is 90°, and you can write the equation $(6k - 8) + (7k - 45) = 90$. This simplifies to $13k - 53 = 90$ and then to $13k = 143$. Divide both sides by 13 to see that $k = 11$. Grid in **11**.

11. B

Difficulty: Hard

Getting to the Answer: Since *BA* and *ED* are both perpendicular to *AC*, they are parallel, and triangles *ABC* and *DEC* are similar. If $\tan \angle B = \frac{AC}{AB}$, then $2.4 = \frac{AC}{5}$, and $AC = 12$. Triangle *ABC* is a 5:12:13 right triangle, and the length of *BC* is 13. (If you didn't recall Pythagorean triple, you could have calculated the hypotenuse using the Pythagorean theorem.) Given that $AD = 3$ and $AC = 12$, it follows that $DC = 12 - 3 = 9$, and the ratio of the side lengths of $\triangle DEC$ to $\triangle ABC$ is 9:12, which simplifies to 3:4. Because this ratio is the same for all sides, *BE* is $\frac{1}{4}$ the length of *BC*, or $\frac{1}{4}(13) = 3\frac{1}{4}$. **(B)** is correct.