CHAPTER 11

Exponents, Radicals, Polynomials, and Rational Expressions

LEARNING OBJECTIVES

After completing this chapter, you will be able to:

- Apply exponent rules
- Apply radical rules
- Add, subtract, multiply, and factor polynomials
- Divide polynomials
- Define root, solution, zero, and *x*-intercept and identify them on the graph of a nonlinear function
- Determine whether the growth or decay described in a question is linear or exponential
- Apply the linear and exponential equations to answer growth and decay questions
- Simplify rational expressions
- Isolate a variable in a rational equation

25/600 SmartPoints® (Medium Yield)

Math

How Much Do You Know?

Directions: Try the questions that follow. Show your work so that you can compare your solutions to the ones found in the Check Your Work section immediately after this question set. The "Category" heading in the explanation for each question gives the title of the lesson that covers how to solve it. If you answered the question(s) for a given lesson correctly, and if your scratchwork looks like ours, you may be able to move quickly through that lesson. If you answered incorrectly or used a different approach, you may want to take your time on that lesson.

1. Which expression is equivalent to $2(-4j^3k^{-4})^{-3}$?

A) $-\dfrac{k^{12}}{512j^9}$

B) $-\dfrac{k^{12}}{32j^9}$

C) $-\dfrac{j^9}{32k^{12}}$

D) $-\dfrac{k^{12}}{128j^9}$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

2. The formula above was created by Italian scientist Galileo Galilei in the early 1600s to demonstrate that the time it takes for a pendulum to complete a swing—called its period, T—can be found using only the length of the pendulum, L, and the force of gravity, g. He proved that the mass of the pendulum did not affect its period. Based on the equation above, which of the following equations could be used to find the length of the pendulum given its period?

A) $L = \dfrac{gT}{2\pi}$

B) $L = \dfrac{gT^2}{4\pi^2}$

C) $L = \dfrac{T^2}{4\pi^2 g}$

D) $L = \dfrac{g}{4\pi^2 T^2}$

3. Which of the following represents $\dfrac{\sqrt[6]{x^{10}y^{12}}}{\sqrt[3]{x^5y^6}}$ written in simplest form, given that $x > 0$ and $y > 0$?

A) 1

B) 2

C) $x^2y^3\sqrt{x}$

D) $xy^2\sqrt[3]{x^2}$

4. What is the result when $4x^3 - 5x^2 + x - 3$ is divided by $x - 2$?

A) $4x + 3 + \dfrac{11}{x-2}$

B) $4x^2 + 3x - 6$

C) $4x^2 + 3x + 18$

D) $4x^2 + 3x + 7 + \dfrac{11}{x-2}$

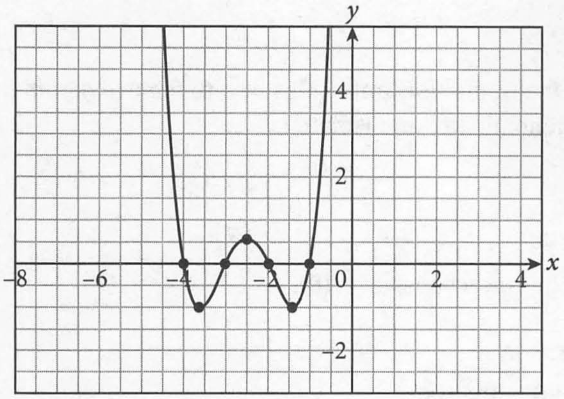
5. The function f is a parabolic function that intersects the x-axis. Which of the following statements must be true?

A) The function has at least one real root.

B) The function has no real roots.

C) The function intersects the positive y-axis.

D) The function has two zeros.



6. The function $f(x) = (x + 1)(x + 2)(x + 3)(x + 4)$ is graphed above. If k is a constant such that $g(x) = k$ and the system of functions f and g have exactly two real solutions, which of the following could be a value of k?

A) -2

B) 0

C) 0.5

D) 1

7. In the equation $ax^4 + bx^3 + cx^2 - dx = 0$, a, b, c, and d are constants. If the equation crosses the x-axis at 0, -2, 3, and 5, which of the following is a factor of $ax^4 + bx^3 + cx^2 - dx$?

A) $x - 2$

B) $x + 3$

C) $x - 5$

D) $x + 5$

Age (months)	Height (centimeters)
15	80
18	82.5
21	85
24	87.5
27	90

8. The growth of a young child is given in the chart above. If t represents the number of months after 15 and $f(t)$ represents the child's height, which of the following equations is the best model for the data in the age range shown?

A) $f(t) = 80t + 3$

B) $f(t) = \frac{5}{6}t + 80$

C) $f(t) = 80\left(\frac{5}{6}\right)^t$

D) $f(t) = \frac{5}{6}(80)^t$

$$\frac{8x}{3(x-5)} + \frac{2x}{3x-15} = \frac{50}{3(x-5)}$$

9. What value(s) of x satisfy the equation above?

A) 0

B) 5

C) No solution

D) Any value such that $x \neq 5$

10. If $\frac{6}{2x-3} + 6a = \frac{10}{2x-3} + 4b$ and $3a - 2b = 2$, what is the value of x?

A) $\frac{1}{2}$

B) 2

C) No solution

D) The value cannot be determined from the information given.

Check Your Work

1. B

Difficulty: Easy

Category: Exponents

Getting to the Answer: Move the expression in parentheses to the denominator to make the sign of the exponent outside positive; do not change the signs of the exponents inside the parentheses. Next, distribute the exponent as usual. Divide the 2 into −64, and move k^{-12} back to the numerator and change the sign of its exponent. Work for these steps is shown below:

$$\begin{aligned} 2\left(-4j^3k^{-4}\right)^{-3} &= \frac{2}{\left(-4j^3k^{-4}\right)^3} \\ &= \frac{2}{(-4)^3\left(j^3\right)^3\left(k^{-4}\right)^3} \\ &= \frac{\cancel{2}}{-\cancel{64}j^9k^{-12}} \\ &= -\frac{k^{12}}{32j^9} \end{aligned}$$

Choice **(B)** is the correct answer.

2. B

Difficulty: Medium

Category: Radicals

Getting to the Answer: The question asks you to solve the equation for L. Use inverse operations to accomplish the task. Divide both sides of the equation by 2π and then square both sides. You'll need to apply the exponent to all the terms on the left side of the equation, including the π:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{L}{g}}$$

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{L}{g}}\right)^2$$

$$\frac{T^2}{4\pi^2} = \frac{L}{g}$$

Finally, multiply both sides by g to remove g from the denominator and isolate L:

$$L = \frac{gT^2}{4\pi^2}$$

The correct answer is **(B)**.

3. A

Difficulty: Hard

Category: Radicals

Getting to the Answer: You can't simplify an expression that has different roots (in this case, sixth and third), so rewrite the expression with fraction exponents first, and then use exponent rules to simplify it. The rule for fraction exponents is "power over root"; use this to rewrite the expression:

$$\frac{\sqrt[6]{x^{10}y^{12}}}{\sqrt[3]{x^5y^6}} = \frac{x^{\frac{10}{6}}y^{\frac{12}{6}}}{x^{\frac{5}{3}}y^{\frac{6}{3}}}$$

When dividing like bases, subtract the exponents. Find common denominators as needed:

$$x^{\frac{10}{6}-\frac{5}{3}}y^{\frac{12}{6}-\frac{6}{3}} = x^{\frac{10}{6}-\frac{10}{6}}y^{\frac{12}{6}-\frac{12}{6}} = x^0y^0$$

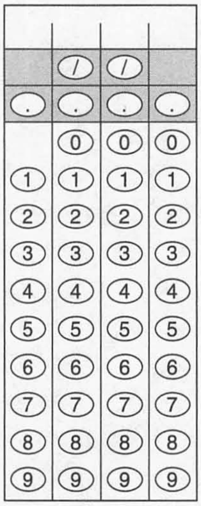
Any number raised to the zero power becomes 1, so the expression becomes $1 \times 1 = 1$. Choice **(A)** is correct.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q1, how can you get all three bases to have the same value?

1. What is the value of $\frac{3^5 \times 27^3}{81^3}$?



HINT: For Q2, look for common factors in the numerator and denominator.

$$\frac{18x^4 + 27x^3 - 36x^2}{9x^2}$$

2. If $x \neq 0$, which of the following is equivalent to the expression above?

A) $2x^2 + 3x - 4$

B) $2x^2 + 3x - 6$

C) $2x^4 + 3x^3 - 4x^2$

D) $2x^6 + 3x^5 - 4x^4$

3. Human blood contains three primary cell types: red blood cells (RBC), white blood cells (WBC), and platelets. In an adult male, a single microliter (1×10^{-3} milliliters) of blood contains approximately 5.4×10^6 RBC, 7.5×10^3 WBC, and 3.5×10^5 platelets on average. What percentage of an adult male's total blood cell count is comprised of red blood cells?

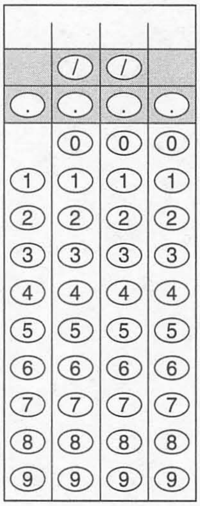
A) 1.30%

B) 6.21%

C) 60.79%

D) 93.79%

4. If $n^3 = -8$, what is the value of $\frac{\left(n^2\right)^3}{\frac{1}{n^2}}$?



HINT: For Q5, how can you get rid of the fraction on the left side?

$$\frac{x^{5r}}{x^{3r-2s}} = x^t$$

5. If $r + s = 6$, what is the value of t in the equation above?

A) 6

B) 12

C) 18

D) 30

Radicals

LEARNING OBJECTIVE

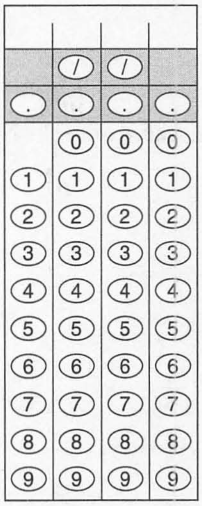
After this lesson, you will be able to:

- Apply radical rules

To answer a question like this:

$$\frac{\sqrt[3]{x} \cdot x^{\frac{5}{2}} \cdot x}{\sqrt{x}}$$

If x^n is the simplified form of the expression above, what is the value of n ?



Drill answers:

a. $3^4 = 3 \times 3 \times 3 \times 3 = 81$

b. $(-5)^3 = (-5) \times (-5) \times (-5) = -125$

c. $4^2 \times 2^{-4} = \frac{16}{16} = 1$

d. $\frac{2^4}{2^3} = 2^1 = 2$

e. $\left(\frac{1}{3}\right)^{-2} = \left(\frac{3}{1}\right)^2 = 9$

f. $\left(2^2\right)^3 = 2^{2\times3} = 2^6 = 64$

g. $(7x)^2 = 49x^2$

h. $\left(-\frac{1}{2}\right)^{-2} = (-2)^2 = 4$

i. $\left(a^2\right)^5 = a^{10}$

j. $\left(b^3\right)^{-6} = b^{-18} = \frac{1}{b^{18}}$

Math

Polynomial Division

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Divide polynomials

To answer a question like this:

Which of the following is equivalent to $\frac{x^2+3x+7}{x+4}$?

A) $\frac{3+7}{4}$

B) $x+\frac{3}{4}$

C) $3+\frac{7}{x+4}$

D) $x-1+\frac{11}{x+4}$

You need to know this:

To divide polynomials, you can use an approach called **polynomial long division.** This process is similar to ordinary long division, except that you use polynomials instead of numbers. In the process described below, the *dividend* is the polynomial to be divided, the *divisor* is the polynomial you are dividing the dividend by, and the *quotient* is the result of the division.

You need to do this:

- Start with the dividend arranged so that the powers are in descending order, for example: x^4+x^3+x+1. If any terms are missing, put in zeros, like this: $x^4+x^3+0x^2+x+1$. Write the problem using a long division sign.
- Divide the first term of the dividend by the first term of the divisor to yield the first term of the quotient.
- Multiply the divisor by the first term of the quotient.
- Subtract the product you got in the last step from the dividend, then bring down the next term, just as you would in ordinary long division. Use the result as the new dividend.
- Repeat the process until you arrive at the remainder.

Math

Explanation:

To divide $x^2 + 3x + 7$ by $x + 4$, set up a long division problem:

$$x + 4\overline{\smash{)}\, x^2 + 3x + 7}$$

Start by dividing the first term of the dividend, x^2, by the first term of the divisor, x. Multiply the entire divisor by x and subtract this product from the dividend:

$$\begin{array}{r} x \\ x + 4\overline{\smash{)}\, x^2 + 3x + 7} \\ -\left(x^2 + 4x\right) \\ \hline -x + 7 \end{array}$$

Next, divide the first term of the result of this subtraction, $-x$, by the first term of the divisor, x, to get -1. Repeat the process of multiplying and subtracting:

$$\begin{array}{r} x - 1 \\ x + 4\overline{\smash{)}\, x^2 + 3x + 7} \\ -\left(x^2 + 4x\right) \\ \hline -x + 7 \\ -(-x - 4) \\ \hline +11 \end{array}$$

You're left with a remainder of 11. Put this over the divisor, $x + 4$, and you're done. The result of the division is $x - 1 + \frac{11}{x + 4}$, which is choice **(D)**.

Math

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q16, because $a - 3$ is not a factor of the numerator, you'll have to use polynomial long division.

16. Which of the following is equivalent to $\frac{2a^2 - 5a - 1}{a - 3}$?

A) $2a - 2$

B) $2a + 1 - \frac{2}{a-3}$

C) $2a + \frac{2}{a-3}$

D) $2a + 1 + \frac{2}{a-3}$

HINT: For Q17, if the fraction simplifies to $ax + b$, the denominator divides evenly into the numerator. Does that suggest another approach?

$$\frac{6x^2 + 19x + 10}{2x + 5}$$

17. If $ax + b$ represents the simplified form of the expression above, then what is the value of $a + b$?

A) 2

B) 3

C) 5

D) 6

18. Which of the following is equivalent to $\frac{4x^2 - 6x}{2x + 2}$?

A) $2x - \frac{10}{2x+2}$

B) $2x - 5 + \frac{10}{2x+2}$

C) $2x - 3$

D) $2x + 5 - \frac{10}{2x+2}$

HINT: The quotient (result of division) times the divisor (the denominator) equals the dividend (the numerator). For Q19, stop as soon as you have the value of t.

19. The equation $\frac{36x^2 + 16x - 21}{tx - 4} = -9x + 5 - \frac{1}{tx - 4}$ is true for all values of x for which $x \neq \frac{4}{t}$, where t is a constant. What is the value of t?

A) −20

B) −4

C) 4

D) 12

20. If the polynomial $f(x)$ is evenly divisible by $x - 5$ and the polynomial $g(x) = f(x) + 4$, what is the value of $g(5)$?

A) −4

B) 0

C) 4

D) 9

Math

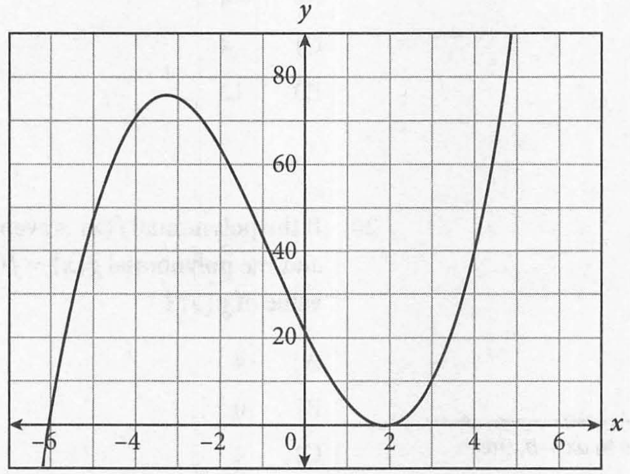
Graphs of Polynomial Functions

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Define root, solution, zero, and *x*-intercept and identify them on the graph of a nonlinear function

To answer a question like this:



Which of the following could be the function whose graph is shown above?

A) $f(x) = (x - 6)(x + 2)^2$

B) $f(x) = (x + 6)(x - 2)^2$

C) $f(x) = 3x + 23$

D) $f(x) = 6x + 23$

You need to know this:

When applied to polynomial functions, the words **root**, **solution**, **zero**, and ***x*-intercept** all mean the same thing: the *x*-values on the function's graph where the function touches or crosses the *x*-axis. You can find the roots of a polynomial function by setting each factor of the polynomial equal to zero. For example, the polynomial function $f(x) = x^2 + x$ factors into $f(x) = x(x+1)$. Set each factor equal to zero to find that $x = 0$ and $x = -1$. These are the function's solutions, also known as zeros. A solution can be represented using the coordinate pair $(x, 0)$.

Note that if a function crosses the *x*-axis, the factor associated with that *x*-intercept will have an odd exponent. If the function touches but does not cross the *x*-axis, the factor associated with that zero will have an even exponent. For example, the function $f(x) = x(x+1)$ will cross the *x*-axis at $x = 0$ and $x = -1$, while the function $f(x) = x^2(x+1)$ will cross the *x*-axis at $x = -1$ but only touch the *x*-axis at $x = 0$.

You need to do this:

- Identify the *x*-values where the function crosses or touches the *x*-axis.
- For each *x*-intercept, change the sign of the *x*-value and add it to the variable *x* to find the associated factor. For example, if the function crosses the *x*-axis at $x = -1$, then the factor associated with that root must be $x + 1$ (since $x + 1 = 0$ will produce the solution $x = -1$).
- Recognize that if the function only touches the *x*-axis without crossing it, the factor must have an even exponent.

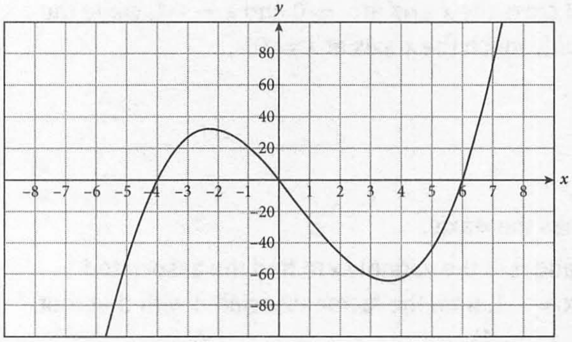
Explanation:

Start by looking at the answer choices. The function is clearly not linear, so rule out (C) and (D). Next, look at the *x*-intercepts on the graph: the function crosses the *x*-axis at $x = -6$ and touches the *x*-axis at $x = 2$. Remember that the *x*-intercepts occur where the factors of the function equal zero. For the *x*-intercepts to be -6 and 2, the factors of the function must be $(x + 6)$ and $(x - 2)$. Because the function touches, but does not cross, the *x*-axis at $x = 2$, the $(x - 2)$ factor must have an even exponent. **(B)** is correct.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q21, set each factor equal to 0 and solve for x to find the x-intercepts.



21. Which of the following could be the equation of the graph above?

A) $y = x^2(x + 4)(x - 6)$

B) $y = x(x + 4)(x - 6)$

C) $y = x^2(x - 4)(x + 6)$

D) $y = x(x - 4)(x + 6)$

x	$h(x)$
−3	6
−1	0
0	−5
2	−8

22. The function h is defined by a polynomial. The table above gives some of the values of x and $h(x)$. Which of the following must be a factor of $h(x)$?

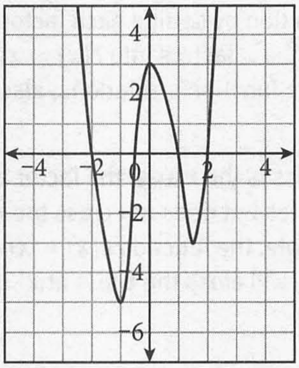
A) $x - 8$

B) $x - 1$

C) $x + 1$

D) $x + 5$

HINT: In Q23, the definition of the b function has a variable in the denominator. What does this tell you about the value of x?



23. The graph of the function $a(x)$ is shown above. If $b(x) = \frac{1}{x}$, which of the following is a true statement about $b(a(x))$?

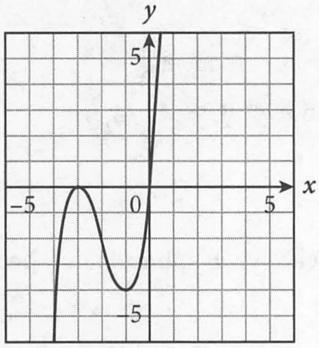
A) $b(a(x))$ is defined for all real numbers.

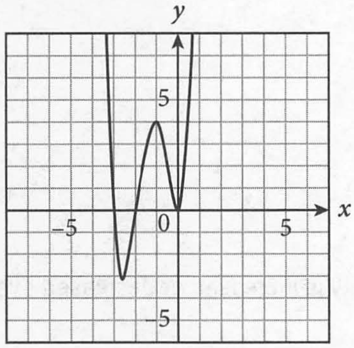
B) $b(a(x))$ is undefined for exactly one real value of x.

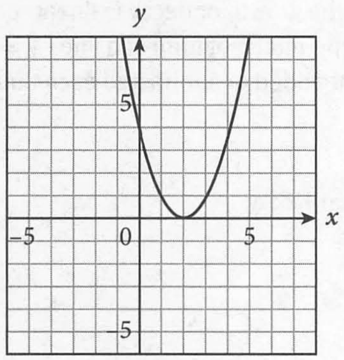
C) $b(a(x))$ is undefined for exactly four real values of x.

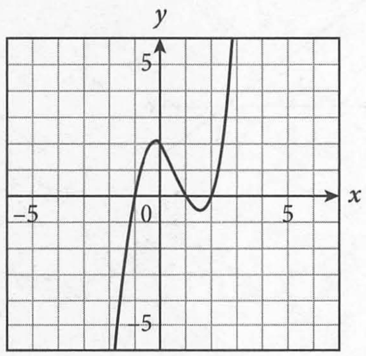
D) $b(a(x))$ is undefined for all real numbers.

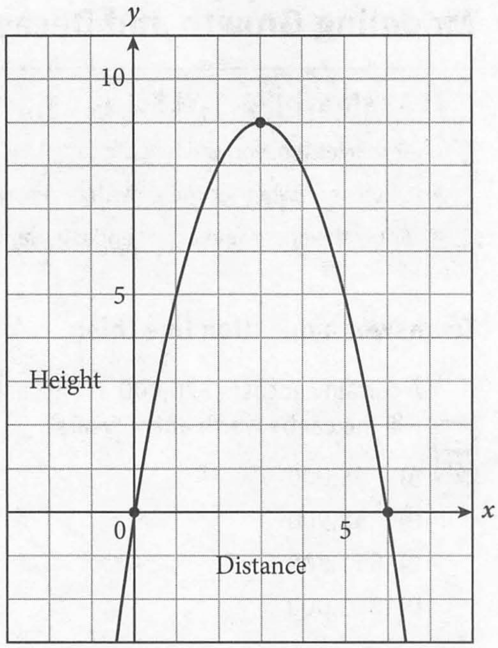
24. If function f has exactly two distinct real zeros, which of the following graphs could be the complete graph of $f(x)$?

A) 

B) 

C) 

D) 



25. The graph of $f(x) = -(x - 3)^2 + 9$ above approximates the trajectory of a water balloon shot from a cannon at ground level. In terms of the trajectory, what information is represented by a root of this function?

A) The maximum height achieved by the balloon
B) The total horizontal distance traveled by the balloon
C) The maximum speed of the balloon
D) The initial acceleration of the balloon

Modeling Growth and Decay

LEARNING OBJECTIVES

After this lesson, you will be able to:

- Determine whether the growth or decay described in a question is linear or exponential
- Apply the linear and exponential equations to answer growth and decay questions

To answer a question like this:

A certain car costs $20,000. If the car loses 15 percent of its value each year, approximately how much will the car be worth after 5 years?

A) $5,000

B) $8,900

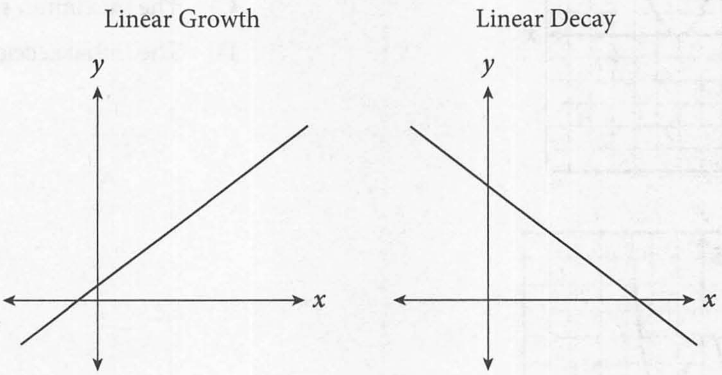
C) $11,200

D) $15,000

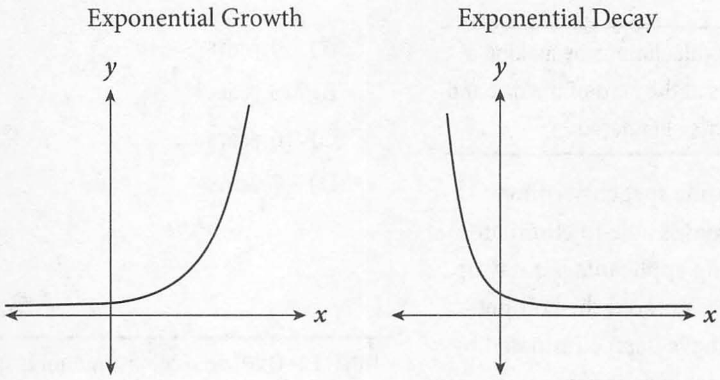
You need to know this:

The terms **growth** and **decay** refer to situations in which some quantity is increased or decreased over time according to a rule:

- If the rule is to add or subtract the same amount each time, then the growth or decay is **linear**. Because the graph of linear growth and decay is a line, you can use the slope-intercept form of a line—$y = mx + b$—to describe it, where b is the starting amount and m is the amount added or subtracted each time x increases by 1.



- If the rule is to multiply or divide by the same amount each time, then the growth or decay is **exponential**. The general form of an exponential function is $y = ab^x$, where a is the y-intercept and b is the amount multiplied or divided each time x increases by 1. Given that $a > 0$ and $b > 1$, when x is positive, the equation describes exponential growth, and when x is negative, the equation describes exponential decay.



- When an exponential growth or decay question gives you a growth rate over time, you can use a modified version of the exponential function, $f(x) = f(0)(1 + r)^t$, where $f(0)$ is the amount at time $t = 0$ and r is the growth rate (or decay rate, if negative) expressed as a decimal.

You need to do this:

- First, determine whether the situation described is linear or exponential. If an amount is added or subtracted each time, then the growth is linear; if the original quantity is multiplied or divided by some amount each time, then the growth is exponential.
- Plug the values from the question into the appropriate equation and solve for the missing quantity.
- When the numbers are manageable, you might be able to avoid using the equations by simply carrying out the operations described. For example, if the question says that an amount doubles each day and asks for the amount after three days, then doubling the initial quantity three times will likely be more efficient than plugging the numbers into the exponential growth equation.

Explanation:

The question says that the car loses value at a certain rate per year, which means the question involves exponential decay. The question gives three pieces of important information: the initial value $f(0) = \$20,000$, the rate $r = -0.15$, and the time $t = 5$. The question asks for the approximate value of the car after 5 years, which is $f(5)$. The rate must be expressed as a decimal, and since the value of the car is decreasing, the rate is also negative. Plug these values into the equation $f(t) = f(0)(1 + r)^t$ and use your calculator to solve:

$$\begin{aligned} f(t) &= 20{,}000(1 - 0.15)^5 \\ &= 20{,}000(0.85)^5 \approx \$8{,}900 \end{aligned}$$

Choice **(B)** is correct.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q26, keep track of your calculations by making a chart with the number of applicants at the start of the day and the number of applicants eliminated.

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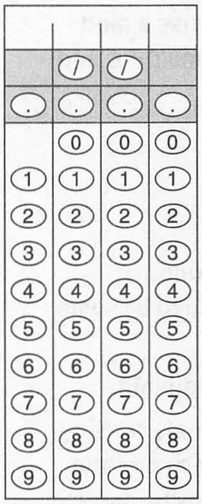
26. In determining the winner of a speech-writing competition, a panel of judges is able to eliminate one-quarter of the remaining applicants per day of deliberations. If 128 students entered the competition, how many applicants have been eliminated by the end of the third day of deliberations?



27. The manager of a health club determines that the club's membership has increased at a rate of 16 percent per year for the past four years. The club currently has 42 members. If this trend continues, how many years will it take for the club's membership to exceed 100 members?

    A) 4 years
    B) 5 years
    C) 6 years
    D) 7 years

HINT: For Q28, no original amount is given. What would be a good number to pick for that amount?

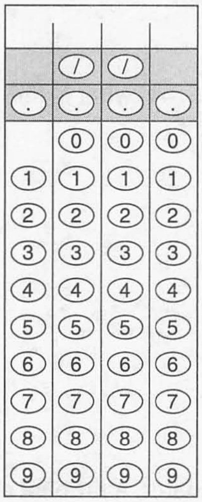
---

28. Radioactive carbon dating can determine how long ago an organism lived by measuring how much of the ^{14}C in the sample has decayed. ^{14}C is an isotope of carbon that has a half-life of 5,600 years. Half-life is the amount of time it takes for half of the original amount to decay. If a sample of a petrified tree contains 6.25 percent of its original ^{14}C, how long ago did the tree die?

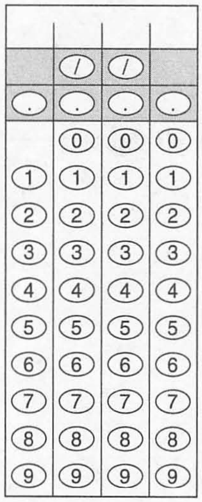
    A) 22,400 years
    B) 28,000 years
    C) 35,000 years
    D) 89,600 years

HINT: For Q29, is she saving more, the same, or less each month? What does that tell you about the function?

29. Penelope receives the same amount of money each month for her allowance. Each month she spends half of her allowance and puts the rest in a piggy bank. On Penelope's 8th birthday, the piggy bank contains $40. If the piggy bank contains $244 after 2 years, what is her monthly allowance? (Ignore the dollar sign when gridding your response.)



30. At a certain bank, money held in account X earns a monthly interest equal to 2 percent of the original investment, while account Y earns a monthly interest equal to 2 percent of the current value of the account. If $500 is invested into each account, what is the positive difference between the value of account X and account Y after three years? (Round your answer to the nearest dollar and ignore the dollar sign when gridding your response.)



Math

Rational Expressions and Equations

LEARNING OBJECTIVES

After this lesson, you will be able to:

- Simplify rational expressions
- Isolate a variable in a rational equation

To answer a question like this:

$$\frac{5y+7}{(y+4)^2}-\frac{5}{(y+4)}$$

If the expression above is equal to $\frac{-b}{(y+4)^2}$, where b is a positive constant and $y \neq -4$, what is the value of b?

A) 4

B) 7

C) 13

D) 27

You need to know this:

A **rational expression** is a ratio expressed as a fraction with a polynomial in the denominator. A **rational equation** is an equation that includes at least one rational expression.

- Factors in a rational expression can be canceled when simplifying, but under no circumstances can you do the same with individual terms. Consider, for instance, the expression $\frac{x^2-x-6}{x^2+5x+6}$. Some test takers will attempt to cancel the x^2, x, and 6 terms to give $\frac{1-1-1}{1+5+1}=\frac{-1}{7}$, which is *never* correct. Instead, factor the numerator and denominator: $\frac{(x+2)(x-3)}{(x+2)(x+3)}$. Cancel the $x+2$ factors to get $\frac{x-3}{x+3}$.
- If a rational expression has a higher-degree numerator than denominator $\left(\text{e.g., } \frac{x^2+3}{1-x}\right)$, it can be simplified using polynomial long division. If a rational expression has a lower-degree numerator than denominator $\left(\text{e.g., } \frac{1-x}{x^2+3}\right)$, it cannot.
- Because rational expressions have polynomial denominators, they will often be undefined for certain values. For example, the expression $\frac{x-4}{x+2}$ is defined for all values of x except -2. This is because when $x=-2$, the denominator of the expression is 0, which would make the expression undefined.
- When solving rational equations, beware of undefined expressions. Take the equation $\frac{1}{x+4}+\frac{1}{x-4}=\frac{8}{(x+4)(x-4)}$, for instance. After multiplying both sides by the common denominator $(x+4)(x-4)$, you have $(x-4)+(x+4)=8$. Solving for x yields $2x=8$, which simplifies to $x=4$. When 4 is substituted for x, however, you get 0 in the denominator of both the second and third terms of the equation. Therefore, this equation is said to have no solution. (A value that causes a denominator to equal 0 is called an extraneous solution.)

Math

You need to do this:

- Find a common denominator.
- Multiply each term by the common denominator and simplify.
- Make sure you haven't found an extraneous solution.

Explanation:

Start by setting the two expressions equal:

$$\frac{5y+7}{(y+4)^2}-\frac{5}{y+4}=\frac{-b}{(y+4)^2}$$

Next, get rid of the fractions. To do this, multiply both sides of the equation by the common denominator, $(y+4)^2$:

$$\left(\frac{5y+7}{(y+4)^2}-\frac{5}{y+4}=\frac{-b}{(y+4)^2}\right)(y+4)^2$$
$$5y+7-5(y+4)=-b$$

Now all that remains is to solve for b:

$$5y+7-5y-20=-b$$
$$-13=-b$$
$$b=13$$

The correct answer is **(C)**.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q31, multiply both sides by a common denominator or cross-multiply. (They are the same thing.)

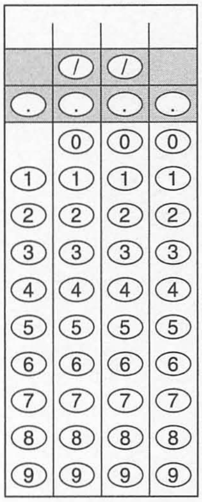
31. Given the equation $\frac{6}{x} = \frac{3}{k+2}$ and the constraints $x \neq 0$ and $k \neq -2$, what is x in terms of k?

A) $x = 2k + 4$
B) $x = 2k + 12$
C) $x = 2k - \frac{1}{4}$
D) $x = \frac{1}{4}k + 12$

HINT: For Q32, how do you add fractions with different denominators?

$$\frac{3a+9}{(a-3)^2} + \frac{-9}{3a-9}$$

32. In the expression above, $(a-3)^2 = 6$. What is the value of the expression?



33. If $a > 6$, which of the following is equivalent to $\dfrac{\frac{2}{a}}{\frac{1}{a-2} + \frac{1}{a-6}}$?

A) $2a^2 - 16a + 24$
B) $a(2a - 8)$
C) $\dfrac{a^2 - 8a + 12}{a^2 - 4a}$
D) $\dfrac{2a - 8}{a^2 - 8a + 12}$

34. If $\frac{5}{x+2} = \frac{2}{x+1} + \frac{1}{2}$ and $x > 1$, what is the value of x?

A) 2
B) 3
C) 6
D) 9

35. If $\frac{16}{7x+4} + A$ is equivalent to $\frac{49x^2}{7x+4}$, what is A in terms of x?

A) $7x + 4$
B) $7x - 4$
C) $49x^2$
D) $49x^2 + 4$

HINT: A common denominator is possible, but messy. Is there another approach to Q36?

$$\frac{c+5}{6c} + \frac{2}{2c-4} = 0$$

36. The equation above is true for all values of c such that $c \neq -6$ and $c \neq 2$. If $c < 0$, what is the value of c?

A) -20
B) -10
C) 1
D) 10

Math

On Test Day

Remember that the SAT doesn't ask you to show your work. If you find the algebra in a question challenging, there is often another way to get to the answer.

Try this question first using algebra and then using the Picking Numbers strategy from chapter 3. Which approach do you find easier? There's no right or wrong answer—just remember your preferred approach and try it first if you see a question like this on test day.

37. The expression $\frac{3x-1}{x-4}$ is equivalent to which of the following?

A) $\frac{1}{2}$

B) $3x - \frac{1}{x-4}$

C) $3 - \frac{11}{x-4}$

D) $3 + \frac{11}{x-4}$

The correct answer and both ways of solving can be found at the end of this chapter.

How Much Have You Learned?

Directions: For testlike practice, give yourself 18 minutes to complete this question set. Be sure to study the explanations, even for questions you got right. They can be found at the end of this chapter.

38. An object launched upwards at an angle has parabolic motion. The height, h, of a projectile at time t is given by the equation $h = \frac{1}{2}at^2 + v_v t + h_0$, where a is the acceleration due to gravity, v_v is the vertical component of the velocity, and h_0 is the initial height. Which of the following equations correctly represents the object's acceleration due to gravity in terms of the other variables?

A) $a = \dfrac{h - v_v t - h_0}{t}$

B) $a = \dfrac{h - v_v t - h_0}{2t^2}$

C) $a = \dfrac{2(h - v_v t - h_0)}{t^2}$

D) $a = t\sqrt{2(h - v_v t - h_0)}$

39. If $\left(16^{3x}\right)\left(32^{x}\right)\left(8^{3x}\right) = \dfrac{\left(4^{6x}\right)\left(32^{3x}\right)}{4}$, what is the value of x?

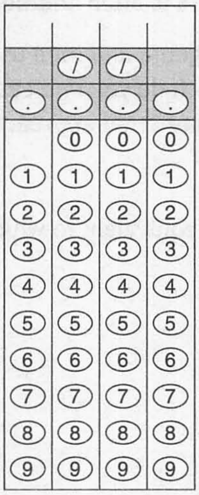
A) −2

B) −1

C) 1

D) 2

40. Given that $\dfrac{y}{\sqrt{x} - 3} = \dfrac{\sqrt{x} + 3}{3}$ and $2x + 42 = 9x - 63$, what is the value of y?



$$y = \frac{3x^2 + 7}{x - 3}$$

41. Which of the following expressions is equivalent to y?

A) $3x + 9 - \dfrac{20}{x - 3}$

B) $3x + 9 + \dfrac{34}{x - 3}$

C) $3x + 43$

D) $3x^2 + \dfrac{9}{x - 3}$

$$z = 15x^2 + 10xy - 6x - 4y$$

42. For which of the ordered pairs, (x, y), below is $z \neq 0$?

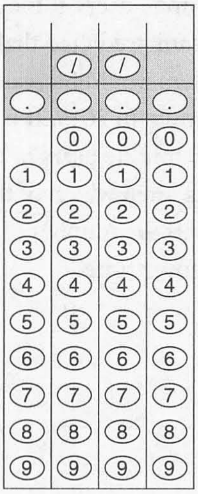
A) $(-3, 2)$

B) $(-2, 3)$

C) $\left(\frac{2}{5}, 0\right)$

D) $\left(\frac{2}{5}, 10\right)$

43. $\sqrt{27^{\frac{2}{3}} + 128^{\frac{4}{7}}} =$



44. Which of the following is equivalent to $\dfrac{4x^2 - 8}{2x + 3}$?

A) $2x - 3 + \dfrac{1}{2x + 3}$

B) $2x - 2$

C) $2x + 3 - \dfrac{1}{2x + 3}$

D) $2x + 4$

$$g(x) = \frac{2}{2x^3 - 12x^2 - 14x}$$

45. For which of the following values of x is the function $g(x)$ defined?

A) -1

B) 0

C) 1

D) 7

$$x^3 + 4 = 3x^2 - 7x + 25$$

46. For what real value of x is the above equation valid?

A) 0

B) 3

C) 4

D) 7

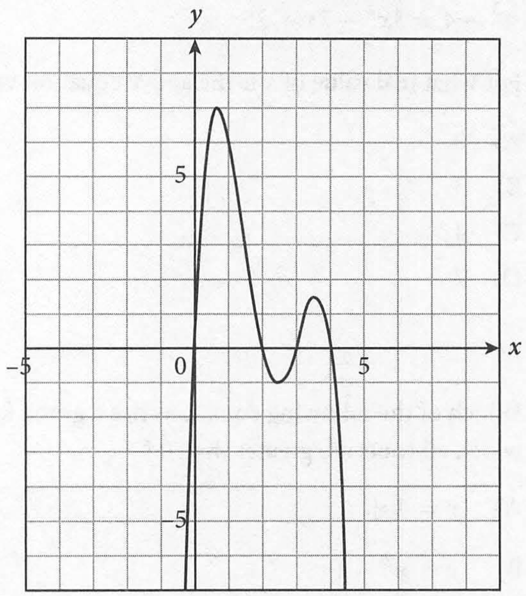
47. Which of the following equations has a graph for which all roots are greater than 0 ?

A) $y = 4|x|$

B) $y = x^2 - 4$

C) $y = (x - 2)^2$

D) $y = x(x - 2)^2$



48. The graph of the function $f(x) = -x(x - 4)(x - 3)(x - 2)$ is shown above. If $f(x) = 0$, how many real solutions exist?

A) 0
B) 2
C) 3
D) 4

49.  A marketing team conducted a study on the use of smartphones. In a certain metropolitan area, there were 1.6 million smartphone users at the end of 2018. The marketing team predicted that the number of smartphone users would increase by 35 percent each year beginning in 2019. If y represents the number of smartphone users in this metropolitan area after x years, then which of the following equations best models the number of smartphone users in this area over time?

A) $y = 1{,}600{,}000(1.35)^x$
B) $y = 1{,}600{,}000(35)^x$
C) $y = 35x + 1{,}600{,}000$
D) $y = 1.35x + 1{,}600{,}000$

Math

Reflect

Directions: Take a few minutes to recall what you've learned and what you've been practicing in this chapter. Consider the following questions, jot down your best answer for each one, and then compare your reflections to the expert responses on the following page. Use your level of confidence to determine what to do next.

How do you multiply two polynomials?

What are the zeros of a polynomial function?

Describe the difference between linear and exponential growth.

Name two ways to simplify rational expressions.

Expert Responses

How do you multiply two polynomials?

Distribute each term in the first set of parentheses to each term in the second set, then combine like terms.

What are the zeros of a polynomial function?

The zeros are the points at which the value of the function (i.e., the y*-value) is zero. Think of the zeros as the* x*-intercepts of the function. The words "roots" and "solutions" mean the same thing.*

Describe the difference between linear and exponential growth.

Linear growth is modeled by a linear function, with the equation $y = mx + b$*, such that the slope of the line is positive. You're essentially adding the same amount over and over again. (In linear decay, the slope of the line is negative, and you're subtracting the same amount over and over again.)*

Exponential growth is modeled by an exponential function in the form $y = ab^x$*, where* a *is the* y*-intercept and* b *is the amount multiplied each time, over and over. The slope of an exponential growth function steepens with increasing* x*-values. (In exponential decay,* b *is the amount divided by each time, and the slope starts out steeply negative and flattens with increasing* x*-values.)*

Name two ways to simplify rational expressions.

One way is to use polynomial long division, though this will work only if the higher-order polynomial is in the numerator. Another way is to cancel factors that appear in both the numerator and denominator.

Next Steps

If you answered most questions correctly in the "How Much Have You Learned?" section, and if your responses to the Reflect questions were similar to those of the SAT expert, then consider exponents and polynomial functions an area of strength and move on to the next chapter. Come back to this topic periodically to prevent yourself from getting rusty.

If you don't yet feel confident, review those parts of this chapter that you have not yet mastered. In particular, review the lessons covering Exponents, Modeling Growth and Decay, and Rational Expressions and Equations as these are high-yield topics on the SAT. Then try the questions you missed again. As always, be sure to review the explanations closely.

Answers and Explanations

1. 9

Difficulty: Medium

Getting to the Answer: You are not allowed to use your calculator on this question, so look for ways to rewrite the larger numbers as the same base. Memorizing the values of small integers up to the fourth or fifth power will help you see these patterns: $27 = 3^3$ and $81 = 3^4$. Now, rewrite the expression as $\frac{3^5 \times (3^3)^3}{(3^4)^3}$, which becomes $\frac{3^5 \times 3^9}{3^{12}} = \frac{3^{14}}{3^{12}}$. Subtract the exponents to get 3^2, which equals **9**, the correct answer.

2. A

Difficulty: Easy

Getting to the Answer: Find the greatest common factor (GCF) of both the numerator and the denominator, which in this question happens to be the denominator. Factor out the GCF, $9x^2$, from the numerator and denominator and then cancel what you can:

$$\begin{aligned}\frac{18x^4 + 27x^3 - 36x^2}{9x^2} &= \frac{\cancel{9x^2}\left(2x^2 + 3x - 4\right)}{\cancel{9x^2}}\\ &= 2x^2 + 3x - 4\end{aligned}$$

This matches **(A)**. As an alternate method, you could split the expression up and reduce each term, one at a time:

$$\begin{aligned}\frac{18x^4 + 27x^3 - 36x^2}{9x^2} &= \frac{18x^4}{9x^2} + \frac{27x^3}{9x^2} - \frac{36x^2}{9x^2}\\ &= 2x^2 + 3x - 4\end{aligned}$$

3. D

Difficulty: Medium

Getting to the Answer: Start by setting up a ratio that compares RBC count to total blood cell count. Manipulate the quantities to make all the exponents the same (to convert 7.5×10^3 to a product of 10^6 and another number, move the decimal point in 7.5 three places to the left and write "$\times 10^6$" after it), factor out 10^6, and then add the quantities in parentheses together. Once there, you can use exponent rules to simplify your equation. Divide through and multiply by 100 to get the RBC component as a percentage:

$$\begin{aligned}\text{RBC} &= \frac{5.4 \times 10^6}{5.4 \times 10^6 + 7.5 \times 10^3 + 3.5 \times 10^5}\\ &= \frac{5.4 \times \cancel{10^6}}{5.4 \times \cancel{10^6} + 0.0075 \times \cancel{10^6} + 0.35 \times \cancel{10^6}}\\ &= \frac{5.4 \times \cancel{10^6}}{\cancel{10^6}\,(5.4 + 0.0075 + 0.35)}\\ &= \frac{5.4}{5.7575}\end{aligned}$$

Note that the answer choices are, for the most part, far apart. Because 5.4 is relatively close to 5.7575, you can conclude with confidence that the correct answer is likely close to 100%. Therefore, **(D)** is the correct answer. If this question is in the calculator section, you can plug the numbers into your calculator to check:

$$\%\text{RBC} = \frac{5.4}{5.7575} \times 100 \approx 93.79\%$$

Choice **(D)** is still correct.

4. 256

Difficulty: Medium

Getting to the Answer: If $n^3 = -8$, then $n = -2$. Simplify the given expression via exponent rules and then plug -2 in for n:

$$\frac{\left((-2)^2\right)^3}{\frac{1}{(-2)^2}} = \frac{4^3}{\frac{1}{4}} = 4^3 \times 4 = 4^4 = 256$$

Grid in **256**.

5. B

Difficulty: Hard

Getting to the Answer: To simplify the division on the left side of the equation, subtract the powers and combine:

$$\begin{aligned}\frac{x^{5r}}{x^{3r-2s}} &= x^{5r-(3r-2s)}\\ &= x^{5r-3r+2s}\\ &= x^{2r+2s}\end{aligned}$$

Note that in the expression $2r + 2s$, it is possible to factor out a 2. Thus, $x^{2r+2s} = x^{2(r+s)}$. The question indicates that $r + s = 6$, so $x^{2(r+s)} = x^{2(6)} = x^{12}$. This is equal to x^t, so $t = 12$. The answer is **(B)**.

6. A

Difficulty: Medium

Getting to the Answer: Solve equations containing radical expressions the same way you solve any other equation: isolate the variable using inverse operations. Start by subtracting 8 from both sides of the equation and then multiply by 3. Then, square both sides to remove the radical:

$$8+\frac{\sqrt{2x+29}}{3}=9$$
$$\frac{\sqrt{2x+29}}{3}=1$$
$$\sqrt{2x+29}=3$$
$$2x+29=9$$

Now you have a simple linear equation that you can solve using more inverse operations: subtract 29 and divide by 2 to find that $x=-10$. Be careful—just because the equation started with a radical and the answer is negative, it does not follow that *No solution* is the correct answer. If you plug -10 into the expression under the radical, the result is a positive number, which means -10 is a perfectly valid solution. Therefore, **(A)** is correct.

7. C

Difficulty: Medium

Getting to the Answer: First, solve for x and plug that value into the second equation. Subtract x from both sides of the equation to get $2x=14$, so $x=7$. Plugging this into the second equation gives you $\sqrt{3z^2-11}+2(7)=22$. Thus, $\sqrt{3z^2-11}=8$. Square both sides of this equation and solve for z:

$$3z^2-11=64$$
$$3z^2=75$$
$$z^2=25$$
$$z=\pm 5$$

Since the question specifies that $z>0$, **(C)** is correct.

8. C

Difficulty: Easy

Getting to the Answer: Follow the standard order of operations—deal with the exponent first and then attach the negative sign (because a negative in front of an expression means multiplication by -1). The variable x is being raised to the $\frac{1}{4}$ power, so rewrite the term as a radical expression with 4 as the degree of the root and 1 as the power to which the radicand, x, is being raised:

$$x^{\frac{1}{4}}=\sqrt[4]{x^1}=\sqrt[4]{x}$$

Now attach the negative to arrive at the correct answer, $-\sqrt[4]{x}$, which is **(C)**.

9. 16

Difficulty: Medium

Getting to the Answer: Because this is a no-calculator question, you need to rewrite the exponent in a way that makes it easier to evaluate: use exponent rules to rewrite $\frac{4}{3}$ as a unit fraction raised to a power. Then write the expression in radical form and simplify:

$$8^{\frac{4}{3}}=\left(8^{\frac{1}{3}}\right)^4$$
$$=\left(\sqrt[3]{8}\right)^4$$
$$=2^4$$
$$=2\times2\times2\times2$$
$$=16$$

Grid in **16**.

10. A

Difficulty: Medium

Getting to the Answer: Start by isolating the radical on the left side of the equation by adding 3 to both sides to get $\sqrt{3a+16}=a+2$. Now you can square both sides to get rid of the radical: $3a+16=(a+2)^2=a^2+4a+4$. Since the right side of this equation is a quadratic, set it equal to 0 in order to determine the solutions: $0=a^2+4a+4-(3a+16)=a^2+a-12$. Next, factor the quadratic using reverse FOIL. The two factors of -12 that add up to 1 are -3 and 4, so $(a-3)(a+4)=0$. Thus, a can be either 3 or -4, but the question says $a>0$, so the only permissible value is 3. **(A)** is correct.

11. A

Difficulty: Easy

Getting to the Answer: Add polynomial expressions by combining like terms. Be careful of the signs of each term. It may help to write the sum vertically, lining up the like terms:

$$\begin{array}{r} 6a^2 - 17a - 9 \\ + -5a^2 + 8a - 2 \\ \hline a^2 - 9a - 11 \end{array}$$

The correct choice is **(A)**.

12. D

Difficulty: Medium

Getting to the Answer: First, write the question as a subtraction problem. Pay careful attention to which expression is being subtracted so that you distribute the negative sign correctly:

$$\begin{aligned} &8x^2 + 4x + 10 - \left(3x^3 + 7x - 5\right) \\ &= -3x^3 + 8x^2 - 3x + 15 \end{aligned}$$

This expression matches **(D)**.

13. D

Difficulty: Medium

Getting to the Answer: Multiply each term in the first expression by $\frac{3}{2}$ and each term in the second expression by -2. Then add the two polynomials by writing them vertically and combining like terms. You'll have to find a common denominator to combine the x-coefficients and to combine the constant terms:

$$\frac{3}{2}A = \frac{3}{2}\left(4x^2 + 7x - 1\right) = 6x^2 + \frac{21}{2}x - \frac{3}{2}$$

$$-2B = -2\left(-x^2 - 5x + 3\right) = 2x^2 + 10x - 6$$

$$\begin{array}{r} 6x^2 + \frac{21}{2}x - \frac{3}{2} \\ + 2x^2 + \frac{20}{2}x - \frac{12}{2} \\ \hline 8x^2 + \frac{41}{2}x - \frac{15}{2} \end{array}$$

This means **(D)** is correct. Notice that if you are simplifying the expression from left to right, after you find the x^2-coefficient, you can eliminate (A) and (B). After you find the x-coefficient, you can eliminate (C) and stop your work.

14. C

Difficulty: Hard

Getting to the Answer: In order to solve the equation, move all the terms to one side of the equation and set them equal to 0, then factor the expression. Thus, the given equation becomes $x^3 + x^2 - 9x - 9 = 0$. Think of this as two pairs of terms, $(x^3 + x^2)$ and $(-9x - 9)$. The first pair of terms share a common factor of x^2, so they can be written as $x^2(x + 1)$. The second pair share the common factor of -9, so they are equivalent to $-9(x + 1)$. So, the equation becomes $x^2(x + 1) - 9(x + 1) = 0$. Now, factor out the $(x + 1)$ term: $(x^2 - 9)(x + 1) = 0$.

In order for the product of two terms to be 0, either one or both must be 0. If $x^2 - 9 = 0$, then $x^2 = 9$ and $x = \pm 3$. Eliminate (A) and (D). If $x + 1 = 0$, then $x = -1$. Eliminate (B), so **(C)** is correct. You could also answer the question using Backsolving by plugging in each answer choice until you found the value for x that did *not* satisfy the equation.

15. C

Difficulty: Medium

Strategic Advice: To multiply two polynomials, multiply each term in the first factor by each term in the second factor, then combine like terms.

Getting to the Answer: Multiply each part of the trinomial expression by each part of the binomial one piece at a time and then combine like terms:

$$\begin{aligned} &\left(2x^2 + 3x - 4\right)(3x + 2) \\ &= 2x^2(3x + 2) + 3x(3x + 2) - 4(3x + 2) \\ &= 6x^3 + 4x^2 + 9x^2 + 6x - 12x - 8 \\ &= 6x^3 + 13x^2 - 6x - 8 \end{aligned}$$

Because a represents the coefficient of x^2, $a = 13$. Hence, **(C)** is correct.

Math

16. D

Difficulty: Medium

Getting to the Answer: Use polynomial long division to simplify the expression:

$$\begin{array}{r} 2a+1 \\ a-3\overline{)2a^2-5a-1} \\ -(2a^2-6a) \\ \hline a-1 \\ -(a-3) \\ \hline 2 \end{array}$$

The quotient is $2a+1$ and the remainder is 2, which will be divided by the divisor in the final answer: $2a+1+\frac{2}{a-3}$. Thus, **(D)** is correct.

17. C

Difficulty: Hard

Getting to the Answer: A fraction is the same as division, so you can use polynomial long division to simplify the expression:

$$\begin{array}{r} 3x+2 \\ 2x+5\overline{)6x^2+19x+10} \\ -(6x^2+15x) \\ \hline 4x+10 \\ -(4x+10) \\ \hline 0 \end{array}$$

The simplified expression is $3x+2$, so $a+b=3+2=5$, which is **(C)**. As an alternate approach, you could factor the numerator of the expression and cancel common factors:

$$\frac{6x^2+19x+10}{2x+5}=\frac{\cancel{(2x+5)}(3x+2)}{\cancel{(2x+5)}}=3x+2$$

18. B

Difficulty: Medium

Getting to the Answer: Use polynomial long division to simplify the expression:

$$\begin{array}{r} 2x-5 \\ 2x+2\overline{)4x^2-6x} \\ -(4x^2+4x) \\ \hline -10x \\ -(-10x-10) \\ \hline +10 \end{array}$$

The quotient is $2x-5$ and the remainder is 10. Put the remainder over the divisor and add this to the quotient: $2x-5+\frac{10}{2x+2}$. **(B)** is correct.

19. B

Difficulty: Hard

Getting to the Answer: The question provides the quotient of $-9x+5$ of a division problem and asks you to find the coefficient of the first term of the divisor $tx-4$. Set this up in polynomial long division form to better understand the relationship between t and the other terms:

$$\begin{array}{r} -9x+5 \\ tx-4\overline{)36x^2+16x-21} \end{array}$$

Note that, although the quotient of $-9x+5$ leaves a remainder of $-\frac{1}{tx-4}$, it's not necessary to consider the remainder when determining the value of t. Viewed this way, it becomes apparent that $36x^2 \div tx = -9x$. Multiplying both sides by tx gives you $tx(-9x)=36x^2$; therefore, $t(-9)=36$, so $t=-4$. **(B)** is correct.

20. C

Difficulty: Hard

Getting to the Answer: Because $f(x)$ is divisible by $x-5$, the value $x-5$ must be a factor of $f(x)$. Therefore, you can define $f(x)$ as $(x-5)(n)$, where n is some unknown polynomial. Since $g(x)$ is $f(x)+4$, you can say that $g(x)$ must be $(x-5)(n)+4$.

Thus, $g(5)$ will be $(5-5)(n)+4=0(n)+4=0+4=4$. Therefore, **(C)** is correct.

21. B

Difficulty: Medium

Getting to the Answer: The solutions, or x-intercepts, of a polynomial are the factors of that polynomial. This polynomial has x-intercepts of -4, 0, and 6. The factors that generate those solutions are $(x+4)$, x, and $(x-6)$. Eliminate (C) and (D) because they do not include those three factors. Because the graph *crosses* the x-axis at each x-intercept (rather than merely touching the x-axis), none of the factors can be raised to an even exponent. Therefore, the x^2 term in (A) means it is incorrect. **(B)** is correct.

22. C

Difficulty: Medium

Getting to the Answer: To find the solutions to a polynomial function, factor the polynomial and set each factor equal to 0. The solutions of a function are the x-intercepts, so $h(x)$ or the y-coordinate of the solution must equal 0. From the chart, the only point with $h(x) = 0$ is at $x = -1$. If $x = -1$, the factor that generates that solution is $x + 1 = 0$ because $(-1) + 1 = 0$. **(C)** is correct.

23. C

Difficulty: Hard

Getting to the Answer: Translate the notation: $b(a(x))$ means b of $a(x)$. This tells you to use $a(x)$ as the input for $b(x)$. You can rewrite this as $\frac{1}{a(x)}$, which is the reciprocal of $a(x)$. This new function will be undefined anywhere that $a(x) = 0$ because a denominator of 0 is not permitted. Looking at the graph, you can see that $a(x)$ crosses the x-axis four times, at which point the value of $a(x)$ is 0. Since division by 0 is undefined, **(C)** is correct.

24. A

Difficulty: Easy

Getting to the Answer: The phrase "exactly 2 distinct real zeros" means that the graph must have exactly two different x-intercepts on the graph. An x-intercept is indicated any time that the graph either crosses or touches the x-axis. (B) and (D) have three distinct zeros, and (C) has two zeros, but because the graph only touches the x-axis, they are the same, not distinct. The only graph with exactly two distinct zeros is **(A)**.

25. B

Difficulty: Easy

Getting to the Answer: The keyword "root" in the question stem means that you should examine the places at which the graph intersects the x-axis. Thus, this graph has roots at (0, 0) and (6, 0). The x-axis, according to the graph, represents the distance traveled by the balloon. When $x = 0$, the distance the water balloon has traveled is 0, which is the balloon's starting position. The initial location of the balloon is not an answer choice, so the correct answer must be what the other root represents. When $x = 6$, the balloon's height is 0, which is the end point of the balloon's trajectory. This value, 6, is a root that represents the total horizontal distance traveled. **(B)** is correct.

26. 74

Difficulty: Easy

Strategic Advice: The goal is to find the number of applicants *eliminated* after four days, not the number remaining.

Getting to the Answer: The question describes the decay as the result of removing a certain fraction of the remaining applicants each day. The situation involves repeated division, so this is an example of exponential decay. You could use the exponential decay formula for a given rate, but without a calculator, raising a decimal to an exponent might create time-consuming calculations. Instead, determine how many applicants are eliminated each day and tally them up.

After the first day, the judges eliminate one-fourth of 128, or 32, applicants. This leaves $128 - 32 = 96$ applicants. On the second day, one-fourth of 96, or 24, applicants are eliminated, leaving $96 - 24 = 72$. Finally, on the third day, one-fourth are eliminated again; one-fourth of 72 is 18, so there are $72 - 18 = 54$ applicants remaining. If 54 applicants remain, then $128 - 54 = 74$ applicants have been eliminated. Grid in **74**.

27. C

Difficulty: Medium

Strategic Advice: This question gives you a percent increase per year, so use the exponential growth equation to solve for the number of years. Note that the question provides unnecessary information. It doesn't matter that the trend has been happening for the last four years because the question asks only about the number of years after the present.

Getting to the Answer: Use the formula for exponential growth and plug in the values from the question. The rate is 16%, which as a decimal is 0.16. The rate will remain positive because the question asks about increase, or growth; therefore, $r = 0.16$. The current number of members is 42, so this will be $f(0)$. The goal is at least 100 members, so that will be the output, or $f(t)$. Put it all together:

$$f(t) = f(0)(1 + r))^t$$
$$100 = 42(1 + 0.16)^2$$
$$100 = 42(1.16)^t$$

At this point, Backsolving is the best approach. Plug in the number of years for t. Because the answer choices are in ascending order, try one of the middle options first. You might be able to eliminate more than one choice at a time. Choice (B) is $t = 5$:

$$42(1.16)^5 \approx 88$$

Since (B) is too small, (A) must be as well. Eliminate them both. Unfortunately, 88 is not close enough to 100 to be certain that **(C)** is the correct answer, so test it:

$$42(1.16)^6 \approx 102$$

Six years is enough to put the club over 100 members. **(C)** is correct.

28. A

Difficulty: Hard

Strategy Advice: The term "half-life" signals exponential decay because it implies repeated division by 2. Using the exponential decay formula here would be complicated. Instead, you can use the percentage given in the question, along with the Picking Numbers strategy, to figure out how many half-lives have elapsed.

Getting to the Answer: Instead of providing an actual amount of ^{14}C, this question tells you what percent is left. For questions involving percentages of unknown values, it is often a good idea to pick 100. So, assume that the amount of ^{14}C in the sample when the tree died is 100. (Fortunately, there is no need to worry about the units here.) After one half-life, the amount of ^{14}C is halved to 50. A second half-life leaves 25, a third leaves 12.5, and a fourth leaves 6.25, which is 6.25% of 100. So four half-lives have elapsed. Since each half-life is 5,600 years, the tree died $4 \times 5{,}600$ or 22,400 years ago. Choice **(A)** is correct.

29. 17

Difficulty: Medium

Strategic Advice: The question describes a situation with linear growth since Penelope is adding the same amount of money to her piggy bank each month. Note: the question is asking for her monthly allowance, but she puts in only half that amount each month.

Getting to the Answer: Use the linear growth equation $y = mx + b$. The question gives you the starting amount b (\$40), the final amount y (\$244), and the amount of time x (2 years, which is 24 months). Plug these values into the equation and solve for m, which is the slope, or the rate of change—or in this case, how much Penelope puts in her piggy bank each month:

$$y = mx + b$$
$$244 = m(24) + 40$$
$$24m = 204$$
$$m = 8.5$$

Remember that what she puts in the piggy bank is only half of her allowance, so her total monthly allowance is twice \$8.50. Grid in **17**.

30. 160

Difficulty: Hard

Strategic Advice: This question describes both types of growth. Account X adds a percentage of the original amount, which never changes, so the same amount of money is added each month. Account X grows linearly. Account Y, however, adds a percentage of the current balance, which grows monthly, so account Y grows exponentially.

Getting to the Answer: Account X begins with \$500 (the *y*-intercept, or *b*) and adds 2% of \$500, or $\$500 \times 0.02 = \$10$ (the rate of change, or *m*), each month for 3 years, which is 36 months (the input, or *x*). Plug these values into the linear growth equation to solve for the final value of the account:

$$y = mx + b$$
$$y = 10(36) + 500$$
$$y = 360 + 500 = \$860$$

Account Y begins with \$500 ($f(0)$) and adds 2%, or 0.02, (r) each month for 36 months (t). Plug these values into the exponential growth equation to solve for the final value of the account:

$$f(t) = f(0)(1 + r)^t$$
$$f(t) = 500(1 + 0.02)^{36}$$
$$f(t) = 500(1.02)^{36} \approx \$1{,}019.94$$

The positive difference between the two accounts is therefore $\$1{,}019.94 - \$860 = \$159.94$. Round up, and grid in **160**.

31. A

Difficulty: Medium

Getting to the Answer: There are two variables and only one equation, but because you're asked to solve for one of the variables *in terms of* the other, you solve the same way you would any other equation, by isolating *x* on one side of the equation. Cross-multiplying is a quick route to the solution:

$$\frac{6}{x} = \frac{3}{k+2}$$
$$6(k+2) = 3x$$
$$6k + 12 = 3x$$
$$\frac{6k}{3} + \frac{12}{3} = \frac{3x}{3}$$
$$2k + 4 = x$$

Switch *x* to the left side of the equation and the result matches **(A)**.

32. 3

Difficulty: Hard

Getting to the Answer: Because the expression is adding fractions with different denominators, you'll need to establish a common denominator. Note that the second fraction is divisible by 3, so you can simplify the expression and then create the common denominator:

$$\frac{3a+9}{(a-3)^2} + \frac{-3}{a-3}$$
$$= \frac{3a+9}{(a-3)^2} + \frac{-3}{a-3} \times \frac{a-3}{a-3}$$
$$= \frac{3a+9}{(a-3)^2} + \frac{-3a+9}{(a-3)^2}$$
$$= \frac{18}{(a-3)^2}$$

The question specifies that $(a-3)^2 = 6$, so $\frac{18}{(a-3)^2} = \frac{18}{6} = 3$. Therefore, the expression equals 3. Grid in **3**.

33. C

Difficulty: Medium

Getting to the Answer: The denominator of the expression contains the sum of two fractions that themselves have different denominators, so start by finding a common denominator:

$$\frac{\frac{2}{a}}{\frac{a-6}{(a-2)(a-6)}+\frac{a-2}{(a-2)(a-6)}}=\frac{\frac{2}{a}}{\frac{2a-8}{a^2-8a+12}}$$

Next, multiply the numerator of the expression by the reciprocal of the denominator and simplify:

$$\frac{2}{a}\times\frac{a^2-8a+12}{2a-8}$$
$$=\frac{2\left(a^2-8a+12\right)}{2a^2-8a}$$
$$=\frac{a^2-8a+12}{a^2-4a}$$

This expression matches **(C)**.

34. B

Difficulty: Hard

Getting to the Answer: First, subtract $\frac{2}{x+1}$ from both sides to consolidate the two rational expressions on the same side of the equation. Next, multiply the left side of the equation by $\frac{(x+1)(x+2)}{(x+1)(x+2)}$ to establish a common denominator to enable subtraction of fractions. Next, combine like terms and cross-multiply:

$$\frac{5}{x+2}-\frac{2}{x+1}=\frac{1}{2}$$
$$\frac{5x+5}{x^2+3x+2}-\frac{2x+4}{x^2+3x+2}=\frac{1}{2}$$
$$\frac{3x+1}{x^2+3x+2}=\frac{1}{2}$$
$$6x+2=x^2+3x+2$$

Next, set the equation equal to zero by subtracting $6x+2$ from both sides: $0=x^2-3x$. Now, factor the right side to yield $0=x(x-3)$.

Therefore, either $x=0$ or $x=3$. Since the question specifies that $x>1$, x must equal 3, and **(B)** is correct.

Note that, if you wanted to avoid the algebra, you could backsolve this question by plugging in the answer choices to find which works in the original equation.

35. B

Difficulty: Hard

Getting to the Answer: Because the question states that the expressions are equivalent, set up the equation $\frac{16}{7x+4}+A=\frac{49x^2}{7x+4}$ and solve for A. Start by subtracting the first term from both sides of the equation to isolate A. Then, simplify if possible (usually by cancelling common factors). The denominators of the rational terms are the same, so they can be combined:

$$\frac{16}{7x+4}+A=\frac{49x^2}{7x+4}$$
$$A=\frac{49x^2}{7x+4}-\frac{16}{7x+4}$$
$$A=\frac{49x^2-16}{7x+4}$$
$$A=\frac{\cancel{(7x+4)}(7x-4)}{\cancel{7x+4}}$$
$$A=7x-4$$

The correct choice is **(B)**.

36. B

Difficulty: Medium

Getting to the Answer: While you might be tempted to establish a common denominator in order to add the fractions together, that would be extremely cumbersome. Instead, move the second fraction over to the other side of the equation by subtracting it from both sides. Then cross-multiply to simplify:

$$\frac{c+5}{6c}=\frac{-2}{2c-4}$$
$$(c+5)(2c-4)=-12c$$
$$2c^2+6c-20=-12c$$
$$2c^2+18c-20=0$$
$$c^2+9c-10=0$$
$$(c+10)(c-1)=0$$

Therefore, either $c=-10$ or $c=1$. The question specifies that $c<0$, so c must equal -10. **(B)** is correct.

37. D

Difficulty: Medium

Getting to the Answer: The first thought at seeing this question may be to try to break the expression into two separate fractions or see if some expression can be factored out. Unfortunately, that does not help with simplifying the expression. You'll need polynomial long division if you're going to use algebra:

$$\begin{array}{r} 3 \\ x-4\overline{)3x-1} \\ \underline{-(3x-12)} \\ 11 \end{array}$$

That results in the expression $3+\frac{11}{x-4}$, so the correct answer is **(D).**

Another effective alternative for solving this question is to use Picking Numbers. Choose a small, permissible value like $x=2$ to find a value for the original expression:

$$\frac{3(2)-1}{(2)-4}=\frac{6-1}{-2}=-\frac{5}{2}$$

Now check each answer choice to see which one is equal to $-\frac{5}{2}$ when you plug in $x=2$:

(A): $\frac{3-1}{4}=\frac{2}{4}=\frac{1}{2}$..., eliminate.

(B): $3(2)-\frac{1}{(2)-4}=6-\frac{1}{-2}=$
$6-\left(-\frac{1}{2}\right)=6+\frac{1}{2}=6\frac{1}{2}=\frac{13}{2}$..., eliminate.

(C): $3-\frac{11}{(2)-4}=3-\frac{11}{-2}=3-\left(-\frac{11}{2}\right)=3+\frac{11}{2}=\frac{17}{2}$
..., eliminate.

(D): $3+\frac{11}{(2)-4}=3+\frac{11}{-2}=3+\left(-\frac{11}{2}\right)=-\frac{5}{2}$

38. C

Difficulty: Medium

Category: Rational Expressions and Equations

Getting to the Answer: When solving polynomial equations for one variable, begin by moving all terms that don't contain that variable (in this case *a*) to one side of the equation. Once there, multiply both sides by 2 to eliminate the fraction, and then divide by t^2 to isolate a:

$$h=\frac{1}{2}at^2+v_0t+h_0$$
$$h-v_0t-h_0=\frac{1}{2}at^2$$
$$at^2=2(h-v_0t-h_0)$$
$$a=\frac{2(h-v_0t-h_0)}{t^2}$$

Choice **(C)** is the correct answer.

39. D

Difficulty: Medium

Category: Exponents

Getting to the Answer: In order to combine terms with exponents by multiplication or division, the terms must all have the same base. All the bases in the given equation are powers of 2, so restate that equation with all the terms in base two: $\left(2^4\right)^{3x}\left(2^5\right)^{x}\left(2^3\right)^{3x}=\frac{\left(2^2\right)^{6x}\left(2^5\right)^{3x}}{2^2}$. When a base with an exponent is raised to a power, multiply the exponents. An exponent in a denominator is equivalent to the same exponent in the numerator with the sign flipped, so $(2^{12x})(2^{5x})(2^{9x})=(2^{12x})(2^{15x})(2^{-2})$.

When the numbers with exponents and the same base are multiplied, add the exponents. Thus, the equation further simplifies to $2^{12x+5x+9x}=2^{12x+15x-2}$. Because the base on each side of the equation is the same, the total of the exponents on both sides must be equal. Therefore, $12x+5x+9x=12x+15x-2$. This further simplifies to $26x=27x-2$, so $x=2$. **(D)** is correct.

40. 2

Difficulty: Medium

Category: Radicals

Getting to the Answer: Conveniently, the conjugate of the denominator, $\sqrt{x}-3$, is the numerator of the other fraction, $\sqrt{x}+3$, so cross-multiplying will rationalize the fraction.

Cross-multiplying the first equation yields $(y)(3)=(\sqrt{x}+3)(\sqrt{x}-3)$, which simplifies to $3y=x-9$. (If this is a difficult simplification for you, study FOIL and the classic quadratics in chapter 12.) In the second equation, you are given that $2x+42=9x-63$, so $7x=105$, and $x=15$. Therefore, $3y=15-9=6$, and $y=2$. Grid in **2**.

You could have chosen to solve the second equation for the value of *x*, then substituted the result into the proportion and solved for *y*. Either approach would be equally effective.

41. B

Difficulty: Medium

Category: Polynomial Division

Getting to the Answer: Since there's no common factor in the numerator and denominator, you'll need to resort to using polynomial long division. Notice that the numerator does not have an *x* term, so include 0*x* in the long division so that the fraction can be shown as $x-3\overline{)3x^2+0x+7}$. Next, do the long division:

$$\begin{array}{r} 3x+9 \\ x-3\overline{)3x^2+0x+7} \\ \underline{3x^2-9x} \\ 9x+7 \\ \underline{9x-27} \\ 34 \end{array}$$

The result of the long division is $3x+9$ with a remainder of 34. The remainder is the "leftover" part that hasn't yet been divided by $x-3$, so it can be expressed as $\frac{34}{x-3}$. Thus, **(B)** is correct.

42. A

Difficulty: Hard

Category: Polynomials

Strategic Advice: You could plug the *x*- and *y*-values from each choice into the given equation to see which does not equal zero. However, if you factor the polynomial, you'll save time by avoiding extra calculations.

Getting to the Answer: Look for common factors in the polynomial by groups. The first two terms, $15x^2$ and $10xy$, share the common factor $5x$. The third and fourth terms share a common factor of -2, so you can write the equation as $z=5x(3x+2y)-2(3x+2y)$. Next, factor out the $(3x+2y)$ to regroup this as $z=(5x-2)(3x+2y)$. Since *z* is the product of two factors, if either factor equals 0, then $z=0$.

If $(5x-2)=0$, then $x=\frac{2}{5}$. You can immediately eliminate (C) and (D) because, if $x=\frac{2}{5}$, $z=0$ no matter what the value of *y* might be. If $(3x+2y)=0$, then $x=-\frac{2}{3}y$, as it is in (B). Thus, **(A)** is the only ordered pair for which $z\neq 0$.

43. 5

Difficulty: Hard

Category: Radicals

Getting to the Answer: Remember that the rules for combining terms under a radical are different for addition and multiplication. Although $\sqrt{xy}$ can be written as $\sqrt{x}\times\sqrt{y}$, terms that are added under a radical, such as $\sqrt{x+y}$, must be combined before taking the root. Now, find the value of each term in the radical above, add them, and then take the square root of the sum.

The term, $27^{\frac{2}{3}}$, is the square of the cube root of 27. Since $3\times3\times3=27$, the cube root of 27 is 3, which squared is $3^2=9$. The denominator of the exponent in $128^{\frac{4}{7}}$ quite likely means that 128 is some number to the seventh power. A good place to start would be to try 2 because it is small and easy to raise to high powers. Indeed, you'll find that $2\times2\times2\times2\times2\times2\times2=128$.

So $128^{\frac{4}{7}}=\left(2^7\right)^{\frac{4}{7}}=2^4=16$. Thus,

$\sqrt{27^{\frac{2}{3}}+128^{\frac{4}{7}}}=\sqrt{9+16}=\sqrt{25}=5$. Grid in **5**.

44. A

Difficulty: Medium

Category: Rational Expressions and Equations

Strategic Advice: Since there is no common factor in the numerator and denominator, you might be tempted to dive into polynomial long division. However, if you noticed that the numerator, $4x^2 - 8$, is very close to being the difference of two squares, $4x^2 - 9$, there is a more efficient way to unravel this fraction.

Getting to the Answer: Restate the fraction as $\frac{4x^2 - 9}{2x + 3} + \frac{1}{2x + 3}$. Now factor the numerator and cancel like terms: $\frac{\cancel{(2x + 3)}(2x - 3)}{\cancel{2x + 3}} + \frac{1}{2x + 3}$. **(A)** is correct.

45. C

Difficulty: Medium

Category: Rational Expressions and Equations

Strategic Advice: Recall that a denominator equal to zero results in an undefined value for the fraction. To answer this question, determine which of the choices does *not* result in the denominator being equal to zero.

Getting to the Answer: You could plug each of the answer choices into the denominator to see which three equal zero, or you could factor the denominator first to simplify the identification of those values. Start by factoring out $2x$ to show the denominator as $2x(x^2 - 6x - 7)$. The denominator is the product of the two factors, so if either one is 0, the denominator will be 0. If $2x = 0$, $x = 0$, so eliminate (B) because $x = 0$ makes $g(x)$ undefined.

Next, you could either plug the remaining choices into $x^2 - 6x - 7$ to see which choice does not produce a result of zero or you could factor the expression into $(x - 7)(x + 1)$. Thus, either $x = 7$ or $x = -1$ results in the denominator of $g(x)$ equal to zero. Eliminate (A) and (D); **(C)** is correct. For the record, $g(1) = \frac{2}{-24}$.

46. B

Difficulty: Medium

Category: Polynomials

Getting to the Answer: In order to solve for possible values of x, first group all the terms on one side of the equation and set the other side to 0. Thus, $x^3 + 4 = 3x^2 - 7x + 25$ can be written as $x^3 - 3x^2 + 7x - 21 = 0$. Factor this by grouping the terms into two pairs. The first pair, $x^3 - 3x^2$, contains the common factor x^2, so it is equivalent to $x^2(x - 3)$. The second pair, $7x - 21$, factors to $7(x - 3)$. Thus, another form of the entire equation is $x^2(x-3) + 7(x-3) = 0$. Factor out $(x - 3)$ to get $\left(x^2 + 7\right)(x - 3) = 0$.

If the product of two factors is zero, one or both of the factors must be zero. If $\left(x^2 + 7\right) = 0$, then $x^2 = -7$. Since the square root of a negative number is not a real number, move on to $x - 3 = 0$. In this case, $x = 3$, so **(B)** is correct.

Math

47. C

Difficulty: Medium

Category: Graphs of Polynomial Functions

Strategic Advice: Equations with roots of 0 as well as roots that are less than 0 will be incorrect and can be eliminated. When using elimination, start with the easiest looking choice first, then proceed to the next easiest, and so forth.

Getting to the Answer: Roots on a graph are the points at which $y = 0$. Start by plugging in 0 for y in each choice and solving for x, and then eliminate those choices for which x could be 0 or less. For (A), you would plug in 0 such that $0 = 4|x|$, which is true only for $x = 0$. Eliminate (A). For (B), plug in 0 for y and solve for x:

$$0 = x^2 - 4$$
$$x^2 = 4$$
$$\sqrt{x^2} = \sqrt{4}$$
$$x = \pm 2$$

Since x could equal -2, eliminate (B) as well. Choices (C) and (D) are the products of two factors. If the product of 2 factors is 0, then one of the factors must be 0. In (C), the only value for x that would make the equation equal to 0 is 2, which is greater than 0, so **(C)** is correct. On test day, you would move on, but for the record, in choice (D), $x = 0$ is a root, so (D) is incorrect.

48. D

Difficulty: Medium

Category: Graphs of Polynomial Functions

Getting to the Answer: The notation $f(x) = 0$ is another way of describing the x-axis, so this question is asking how many times the graph intersects or touches the x-axis. This graph crosses the x-axis four times, once each at $x = 0$, $x = 4$, $x = 3$, and $x = 2$, so **(D)** is correct.

49. A

Difficulty: Easy

Category: Modeling Growth and Decay

Getting to the Answer: If the number of smartphone users increases by 35% each year, then the amount of the increase is variable (because it's 35% of a bigger number each time), which means exponential growth. Eliminate (C) and (D). Consider the two remaining equations in terms of the exponential growth function, $f(x) = f(0)(1 + r)^x$. Note that the only difference between the two remaining choices is the r value. Recall that when assembling an exponential growth model, r (the rate) must be in decimal form. Therefore, the number raised to the power of x should be $1 + 0.35$, or 1.35. Choice **(A)** is the only one that fits these criteria.