

CHAPTER 10

Functions

LEARNING OBJECTIVES

After completing this chapter, you will be able to:

- Apply function notation
- Define the domain and range of a function
- Evaluate the output of a function for a given input
- Interpret the graph of a function
- Write a function to describe a rule or data set

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How Much Do You Know?

Directions: Try the questions that follow. Show your work so that you can compare your solutions to the ones found in the Check Your Work section immediately after this question set. The “Category” heading in the explanation for each question gives the title of the lesson that covers how to solve it. If you answered the question(s) for a given lesson correctly, and if your scratch-work looks like ours, you may be able to move quickly through that lesson. If you answered incorrectly or used a different approach, you may want to take your time on that lesson.


$$p(x) = 7x + 4$$

$$s(x) = 7 - p(x)$$

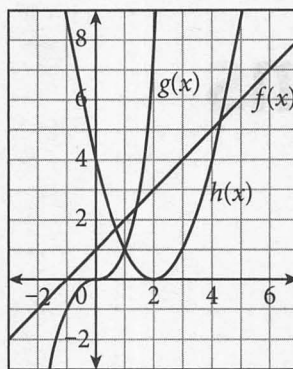
1. What is the value of $s(-1)$?

- A) -3
- B) 4
- C) 10
- D) 17

2. A function is defined by the equation

 $f(x) = \frac{x^2}{4} - 11$. For this function, which of the following domain values corresponds to a range value of 14?

- A) -4
- B) 10
- C) 38
- D) 100



3. In the figure above, what is the value of $h(0) - 3(g(1) - f(2))$?

- A) -2
- B) 5
- C) 10
- D) 12

x	2	3	4	5
$f(x)$	7	13	19	25

4. Some values of the function f are shown in the table above. Which of the following defines f ?



- A) $f(x) = 7x - 1$
- B) $f(x) = 6x - 5$
- C) $f(x) = 5x + 1$
- D) $f(x) = 4x + 5$

5. Briana is writing a 60-page paper for a law school class. She estimates that she will average 45 words per minute while typing. If one page contains approximately 500 words, which of the following correctly estimates the number of pages, p , remaining as a function of the number of minutes, m , that Briana types?

A) $p(m) = 60 - \frac{9m}{100}$

B) $p(m) = \frac{60 - 100}{9m}$

C) $p(m) = 60 - \frac{100}{9m}$

D) $p(m) = \frac{60 - 9m}{100}$

Check Your Work

1. C

Difficulty: Medium

Category: Function Notation

Getting to the Answer: To evaluate a function at any value, substitute that value for the variable in the function definition. The question asks for the value of $s(-1)$, so replace the x in the s function with -1 : $s(-1) = 7 - p(-1)$. To find the value of $p(-1)$, replace the x in the p function with -1 : $p(-1) = 7(-1) + 4 = -3$. Replace $p(-1)$ with its value, -3 , in the s function: $s(-1) = 7 - (-3) = 10$. **(C)** is correct.

2. B

Difficulty: Medium

Category: Function Notation

Getting to the Answer: In this question, you are given a range value (14), which means $f(x) = 14$, and you are asked for the corresponding domain value (x -value). This means you are solving for x , not substituting for x . Set the function equal to 14 and solve using inverse operations:

$$14 = \frac{x^2}{4} - 11$$

$$25 = \frac{x^2}{4}$$

$$100 = x^2$$

$$\pm 10 = x$$

Because -10 is not one of the answer choices, you know the answer is 10. **(B)** is correct.

3. C

Difficulty: Medium

Category: Graphs of Functions

Getting to the Answer: Start with $h(0)$, which means the y -value when $x = 0$. Based on the graph of the h function, $h(0) = 4$. Repeat with $g(1)$, which is 1, and $f(2)$, which is 3. Manipulate these values as instructed: $4 - 3(1 - 3) = 4 + 6 = 10$. Choice **(C)** is correct.

4. B

Difficulty: Easy

Category: Graphs of Functions

Getting to the Answer: Use one of the given points to test the functions. Choose the point that is easiest for you to test. When $x = 2$, $y = 7$, so plug $x = 2$ into the choices and find the one that yields $y = 7$.

Test (A): $f(2) = 7(2) - 1 = 13$. Eliminate (A).

Test (B): $f(2) = 6(2) - 5 = 7$. Keep (B).

Test (C): $f(2) = 5(2) + 1 = 11$. Eliminate (C).

Test (D): $f(2) = 4(2) + 5 = 13$. Eliminate (D).

Only **(B)** remains, and it is correct.

Since all the choices have different slopes, an alternative approach would be to find the slope of the line by using two points.

Use the formula $\text{Slope} = \frac{\text{rise}}{\text{run}}$ to get $\frac{13-7}{3-2} = 6$. **(B)** is correct.

5. A

Difficulty: Hard

Category: Describing Real-Life Situations with Functions

Getting to the Answer: Use the two given rates to determine Briana's typing rate in pages per minute. She types 45 words per minute, which becomes:

$$\frac{45 \text{ words}}{1 \text{ minute}} \times \frac{1 \text{ page}}{500 \text{ words}} = \frac{45 \text{ pages}}{500 \text{ minutes}} = \frac{9 \text{ pages}}{100 \text{ minutes}}$$

Multiplying this rate by m gets you the number of pages typed after m minutes, which can then be subtracted from the starting page count, 60, to get the number of pages Briana has left to type. The function should read $p(m) = 60 - \frac{9m}{100}$, which matches **(A)**.

Function Notation

LEARNING OBJECTIVES

After this lesson, you will be able to:

- Apply function notation
- Define the domain and range of a function
- Evaluate the output of a function for a given input

To answer a question like this:

$$h(x) = \frac{2x + 7}{x - 4}$$

Which of the following must be true about $h(x)$?

- $h(14) = 3.5$
- The domain of $h(x)$ is all real numbers
- $h(x)$ may be positive or negative

- I and II only
- I and III only
- II and III only
- I, II, and III

You need to know this:

A **function** is a rule that generates one unique output for a given input. In function notation, the x -value is the input and the y -value, designated by $f(x)$, is the output. (Note that other letters besides x and f may be used.)

For example, a linear function has the same form as the slope-intercept form of a line; $f(x)$ is equivalent to y :

$$f(x) = mx + b$$

In questions that describe real-life situations, the y -intercept will often be the starting point for the function. You can think of it as $f(0)$, or that value of the function where $x = 0$.

The set of all possible x -values is called the **domain** of the function, while the set of all possible y -values is called the **range**.

You need to do this:

- To find $f(x)$ for some value of x , substitute the concrete value in for the variable and do the arithmetic.
- For questions that ask about the domain of a function, check whether any inputs are not allowed, for example, because they cause division by 0.
- For questions that ask about a function of a function, for example, $g(f(x))$, start on the inside and work your way out.

Explanation:

Check each statement. For the first statement, plug in 14 for x :

$$\frac{2(14) + 7}{14 - 4} = \frac{28 + 7}{10} = \frac{35}{10} = 3.5$$

So the first statement is true. Eliminate choice (C).

For the second statement, you need to determine the set of all permitted x -values for this function. Note that the function will be undefined at $x = 4$ (because at $x = 4$, the denominator would be 0). Thus, 4 is not a permitted x -value, and the domain is not all real numbers. The second statement is false. Eliminate (A) and (D).

By process of elimination, the answer is **(B)**, and on test day, you would stop here. For the record, here's why the third statement is true: you've already established that $h(x) = 3.5$ is a permitted value. If you plug in a smaller value, such as 0, you get: $h(0) = \frac{2(0) + 7}{0 - 4} = \frac{7}{-4} = -\frac{7}{4}$, so $f(x)$ can be negative as well.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

HINT: For Q1, replace every x in the function definition with -2 .

1. If $g(x) = -2x^2 + 7x - 3$, what is the value of $g(-2)$?

- A) -25
- B) -9
- C) -1
- D) 3

2. If $k(x) = 5x + 2$, what is the value of $k(4) - k(1)$?

- A) 15
- B) 17
- C) 19
- D) 21

4. If $p(x) = x^2 - 4x + 8$ and $q(x) = x - 3$, what is the value of $\frac{q(p(5))}{p(q(5))}$?

- A) 0
- B) 0.4
- C) 1
- D) 2.5

n	$f(n)$	$g(n)$
2	11.6	1.5
3	13.9	1
4	16.2	0.5

5. The table above shows some values of the linear functions f and g . If $h(n) = 2 \times f(n) - g(n)$, what is the value of $h(6)$?



- A) 21.3
- B) 35.0
- C) 41.1
- D) 42.1

HINT: For Q3, work from the inside parentheses out.

x	$g(x)$
-6	-3
-3	-2
0	-1
3	0
6	1

x	$h(x)$
0	6
1	-4
2	2
3	0
4	-2

3. Several values for the functions $g(x)$ and $h(x)$ are shown in the tables above. What is the value of $g(h(3))$?

- A) -1
- B) 0
- C) 3
- D) 6

Graphs of Functions

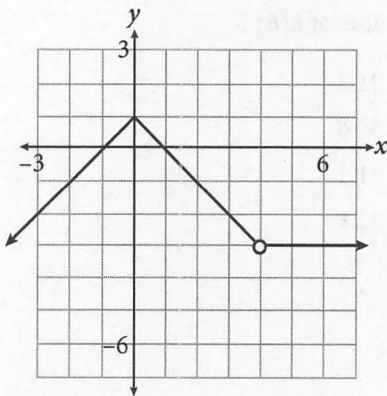
LEARNING OBJECTIVE

After this lesson, you will be able to:

- Interpret the graph of a function

To answer a question like this:

x	$h(x)$
0	-3
1	-2
2	1
3	6
4	13
5	22
6	33

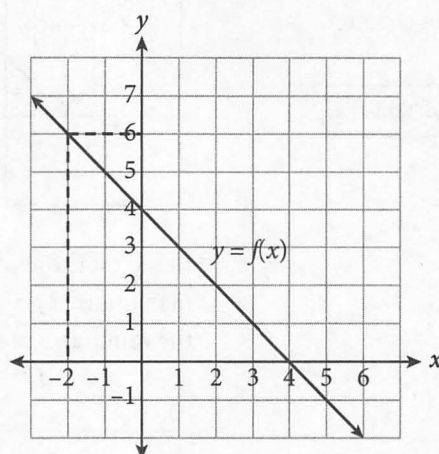


The maximum value of function g , whose graph is shown above, is m . Values for the function h are shown in the table. What is the value of $h(m)$?

- A) -3
- B) -2
- C) 2
- D) 4

You need to know this:

Interpreting graphs of functions is similar to interpreting graphs of equations. For example:



Say the graph above represents the function $f(x)$ and you're asked to find the value of x for which $f(x) = 6$. Because $f(x)$ represents the output value, or range, you can translate this to, "When does the y -value equal 6?" To answer the question, find 6 on the y -axis, then trace over to the function (the line). Read the corresponding x -value: it's -2 , so when $f(x) = 6$, x must be -2 .

The SAT may sometimes ask about a function's **maximum** or **minimum**. These terms mean the greatest and least value of the function, respectively. This graph of $f(x)$ does not have a maximum or minimum, as the arrows on the line indicate that it continues infinitely in both directions. The question above, however, does show a function with a maximum.

You need to do this:

- Treat $f(x)$ as the y -coordinate on a graph.
- Understand that the maximum and minimum refer to a function's greatest and least y -coordinates, respectively.

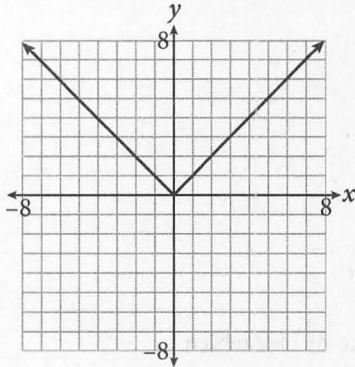
Explanation:

Start by identifying m , which occurs at the apex of the function at $(0, 1)$. The "maximum value of function g " means the greatest y -value, so $m = 1$. Next, use the table to find $h(1)$, which is the y -value when $x = 1$. According to the table, when $x = 1$, $h(x) = -2$. **(B)** is correct.

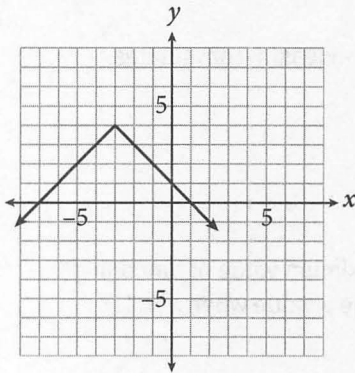
Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

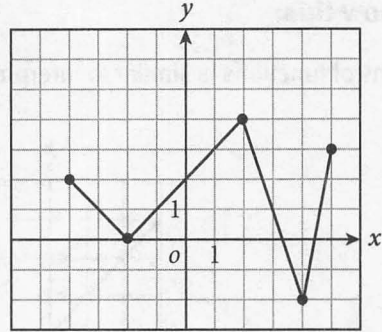
HINT: For Q6, $p(x)$ means the y -value of the function at x .



6. The above figure shows the function $p(x) = |x|$. Which statement about the function is NOT true?
- A) $p(0) = 0$
 - B) $p(-4) = 4$
 - C) $p(4) = -4$
 - D) The domain of $p(x)$ is all real numbers.



7. The graph of $f(x)$ is shown above. Which of the following represents the domain and range of the function?
- A) Domain: $f(x) \geq 4$; range: all real numbers
 - B) Domain: $f(x) \leq 4$; range: all real numbers
 - C) Domain: all real numbers; range: $f(x) \geq 4$
 - D) Domain: all real numbers; range: $f(x) \leq 4$



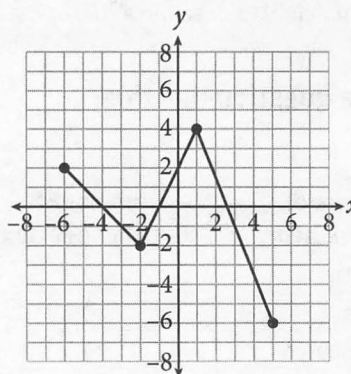
8. Based on the above graph, if the coordinates of the maximum of $f(x)$ are (a, b) and the coordinates of the minimum of $f(x)$ are (c, d) , what is the value of $a + b + c + d$?

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

9. The graph of the linear function f has intercepts at $(c, 0)$ and $(0, d)$ in the xy -plane. If $2c = d$ and $d \neq 0$, which of the following is true about the slope of the graph of f ?

- A) It is positive.
- B) It is negative.
- C) It equals zero.
- D) It is undefined.

HINT: For Q10, which roman numeral statement appears the most often in the answer choices? What advantage would you have if you knew that statement was not part of the correct answer?



10. The complete graph of the function f is shown in the figure above. Which of the following is equal to -1 ?



- I. $f(-4)$
 - II. $f(0)$
 - III. $f(3)$
- A) I and II
 - B) II only
 - C) I, II, and III
 - D) III only

Describing Real-Life Situations with Functions

LEARNING OBJECTIVE

After this lesson, you will be able to:

- Write a function to describe a rule or data set

To answer a question like this:

Type of Ingredient	Number of Cookies per Box	Profit per Box (dollars)
Walnuts	22	1.26
Pecans	20	1.10
Butterscotch	24	1.42
Mint	18	0.94
Macadamias	12	0.46
Hazelnuts	16	0.78

A certain cookie company sells several varieties of chocolate cookies, each with an added ingredient.



The company sells the different varieties in differently sized boxes. The number of cookies per box and the profit per box for the different varieties are shown in the table above. The relationship between the number of cookies per box and the profit, in dollars, that the company makes per box can be represented by a linear function. Which of the following functions correctly represents the relationship?

- A) $p(n) = 0.11n - 0.25$
- B) $p(n) = 0.1n - 0.35$
- C) $p(n) = 0.09n - 0.45$
- D) $p(n) = 0.08n - 0.5$

You need to know this:

Modeling real-life situations using functions is the same as modeling them using equations; the only difference is the function notation and the rule that each input has only one output.

For example, suppose a homeowner wants to determine the cost of installing a certain amount of carpet in her living room. Say that the carpet costs \$0.86 per square foot, the installer charges a \$29 installation fee, and sales tax on the total cost is 7%. Using your algebra and function knowledge, you can describe this situation in which the cost, c , is a function of square footage, f . The equation would be $c = 1.07(0.86f + 29)$. In function notation, this becomes $c(f) = 1.07(0.86f + 29)$, where $c(f)$ is shorthand for “cost as a function of square footage.” The following table summarizes what each piece of the function represents in the scenario.

English	Overall cost	Square footage	Material cost	Installation fee	Sales tax
Math	c	f	$0.86f$	29	1.07

You need to do this:

In word problems involving function notation, translate the math equations exactly as you learned in chapter 4 in the Word Problems lesson, but substitute $f(x)$ for y .

Explanation:

Note that the question asks for the relationship between the number of cookies per box and the profit per box and that the answer choices all start with $p(n)$. Given the context, this translates to the relationship “profit as a function of the number of cookies.” All the choices express a linear relationship, so you can’t rule out any of them on that basis.

There are several approaches you could take to find the correct answer. One would be to recognize that all the choices are in the form $p(n) = kn + b$ (a variation of the slope-intercept form $y = mx + b$) and that you can set up a system of linear equations using the data from any two rows of the table to solve for k and b . That approach would look like this:

$$\begin{array}{r} 1.26 = 22k + b \\ - (1.10 = 20k + b) \\ \hline 0.16 = 2k \\ 0.08 = k \\ \\ 1.10 = 20(.08) + b \\ 1.10 = 1.60 + b \\ b = -0.5 \end{array}$$

Because $k = 0.08$ and $b = -0.5$, the correct function is $p(n) = 0.08n - 0.5$, so **(D)** is correct.

Another approach would be to use two of the pairs of data points from the table to calculate a slope; for example, using the “pecans” and “macadamias” rows would yield $\frac{1.10 - 0.46}{20 - 12} = \frac{0.64}{8} = 0.08$. Because only one answer has a slope of 0.08, you can pick **(D)**.

Finally, you could backsolve. Plug any one of the rows of data from the table into all four answer choices. The second row has the easiest numbers to work with, so use those. You are checking which equation will produce a profit of \$1.10 per box given 20 cookies per box:

- A) $0.11(20) - 0.25 = 1.95 \neq 1.10$
- B) $0.1(20) - 0.35 = 1.65 \neq 1.10$
- C) $0.09(20) - 0.45 = 1.35 \neq 1.10$
- D) $0.08(20) - 0.5 = 1.10$

Again, **(D)** is correct.

Try on Your Own

Directions: Take as much time as you need on these questions. Work carefully and methodically. There will be an opportunity for timed practice at the end of the chapter.

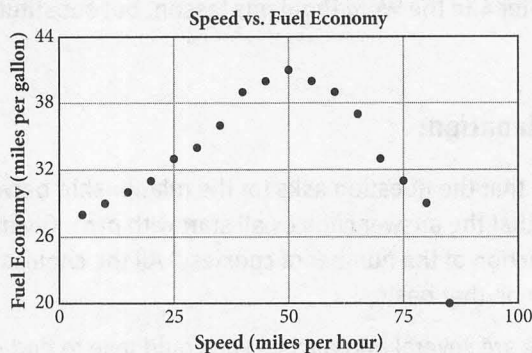
HINT: For Q11, pick the easiest number of days from the chart, plug that into the choices, and eliminate any that don't give you the correct vote count. Repeat if necessary until only one choice is left.

Day	Vote Count
3	21
4	35
5	53
6	75
7	101

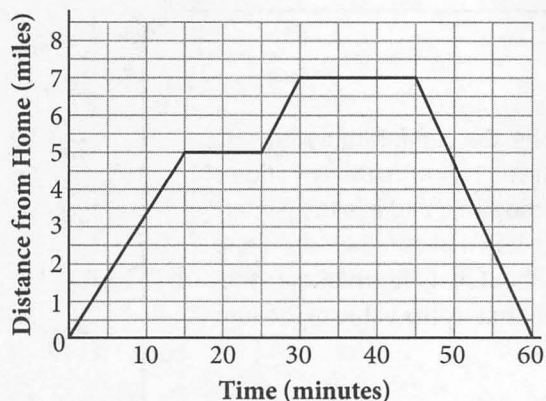
11. Paulo is one of five contest finalists in the running for a year's worth of college book expenses. The winner is the finalist with the highest number of votes on the contest host's website. Paulo recorded his vote total each day of the contest; data for five days are in the table above. Which of the following represents Paulo's vote count, v , as a function of time, t , in days?

- A) $v(t) = 2t^2 + 3$
- B) $v(t) = \frac{t^2}{2} + 3$
- C) $v(t) = 2t^2 + 21$
- D) $v(t) = \frac{t^2}{2} + 21$

HINT: For Q12, the faster the rate of change, the steeper the slope.



12. The graph above shows a compact car's fuel economy as a function of speed. Which of the following is true?
- A) The rate of increase in fuel economy below 50 miles per hour is greater than the rate of decrease in fuel economy above 50 miles per hour.
 - B) The rate of increase in fuel economy below 50 miles per hour is equal to the rate of decrease in fuel economy above 50 miles per hour.
 - C) The rate of increase in fuel economy below 50 miles per hour is less than the rate of decrease in fuel economy above 50 miles per hour.
 - D) Fuel economy peaks at 50 miles per hour, but nothing can be said about the rates of change in fuel economy above and below 50 miles per hour.




13. The graph above shows Carmel's distance from home over a one-hour period, during which time she first went to the library, then went to the grocery store, and then returned home. Which of the following statements must be true?

- A) The grocery store is about 5 miles from Carmel's house.
- B) Carmel traveled a total of 7 miles from the time she left home until she returned.
- C) The grocery store is 7 miles farther from Carmel's house than the library is.
- D) Carmel spent 10 minutes at the library and 15 minutes at the grocery store.

HINT: For Q14, which two readings will be easiest to use to find the number of visitors admitted every 15 minutes?

Time	Total Number of Visitors for the Day
10:10 a.m.	140
12:30 p.m.	420
2:00 p.m.	600
2:50 p.m.	700

14.  The entrance gates at a museum allow a constant number of visitors to enter every 15 minutes. A supervisor records the cumulative number of visitors for the day at various times as shown in the table above. The museum does not admit any visitors after 4:45 p.m. What is the projected total number of visitors for the day, assuming that the same number of visitors are granted entrance each 15-minute period throughout the day?

- A) 810
- B) 895
- C) 930
- D) 960

On Test Day

The SAT likes to test the modeling of real-life situations. Get comfortable with function notation in these questions. Remember that you can write the equation of a line as $y = mx + b$ or as $f(x) = mx + b$, where m is the slope and b is the y -intercept. Both mean the same thing. In the formula using function notation, the slope indicates rate of change. Often, in questions asking about real-life situations, the x variable indicates time. In that case, the y -intercept (that is, the value of the function at $x = 0$, or $f(0)$) indicates the starting point.

15. An environmental agency is working to reduce the amount of plastic that a community discards in the ocean. Currently, the community discards 6.2 million pounds of plastic annually, and the agency's goal is to eliminate that amount by collecting and recycling the plastic. If the agency increases its collection and recycling capacity at a constant rate, and meets its goal at the end of the eighth year, which of the following linear functions, f , could the agency use to model the amount of plastic being added to the ocean t years into the program?

A) $f(t) = -\frac{62}{40}t + 6.2$

B) $f(t) = -\frac{31}{40}t + 6.2$

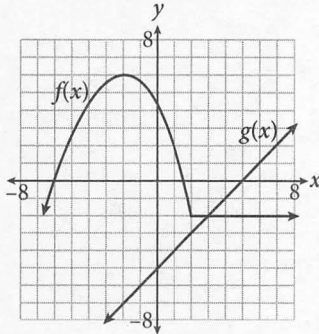
C) $f(t) = \frac{31}{40}t + 6.2$

D) $f(t) = \frac{62}{40}t + 6.2$

The correct answer and explanation can be found at the end of the chapter.

How Much Have You Learned?

Directions: For testlike practice, give yourself 15 minutes to complete this question set. Be sure to study the explanations, even for questions you got right. They can be found at the end of this chapter.



16. Based on the figure above, what is the value of $f(-2) + g(2)$?

- A) -3
- B) 0
- C) 3
- D) 6

17. A company uses the function $P(x) = 150x - x^2$ to determine how much profit the company will make when it sells 150 units of a certain product that sells for x dollars per unit. How much more profit per unit, in dollars, will the company make if it charges \$25 for the product than if it charges \$20? (Ignore the dollar sign when gridding your response.)

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

18. The customer service department of a wireless cellular provider has found that on Wednesdays, the polynomial function $C(t) = -0.0815t^4 + t^3 + 12t$ approximates the number of calls received by any given time, where t represents the number of hours that have passed during the workday. Based on this function, how many calls can be expected by the end of one 10-hour workday?



	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

19. A biologist is studying the effect of pollution on the reproduction of a specific plant. She uses the function $n(p)$ to represent these effects, where p is the number of seeds germinated by the test group of the plant over a given period of time. Which of the following lists could represent a portion of the domain for the biologist's function?

- A) $\{ \dots -150, -100, -50, 0, 50, 100, 150 \dots \}$
- B) $\{-150, -100, -50, 0, 50, 100, 150\}$
- C) $\{0, 0.25, 0.5, 0.75, 1, 1.25, 1.5 \dots\}$
- D) $\{0, 20, 40, 60, 80 \dots\}$

20. If $f(x) = 3 - x$ and $g(x) = \frac{x^2}{2}$, which of the following is NOT in the range of $f(g(x))$?

A) -3
 B) 0
 C) 2
 D) 4

$$g(x) = -3x - 5$$

21. The function g is defined above. What is the value of $g(-4x)$?

A) 7
 B) $-12x - 5$
 C) $12x - 5$
 D) $12x^2 - 20x$

$$r(x) = 3x - 7$$

$$t(x) = 3x + r(x)$$

22. The functions r and t are defined above. What is the value of $t(2)$?

A) -3
 B) -1
 C) 0
 D) 5

$$f(x) = ax^2 + 3x + 5$$

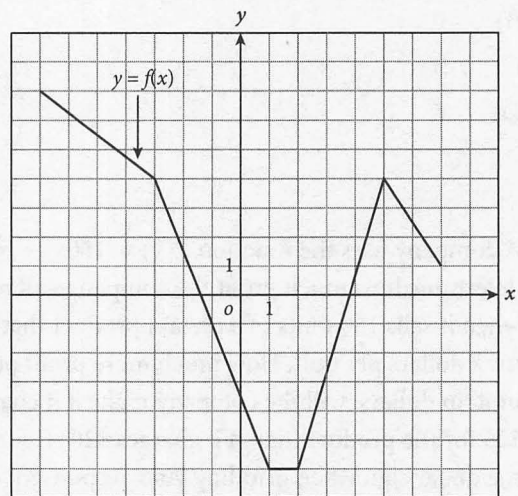
23. The function f is defined above, and $f(3) = -4$. If a is a constant, what is the value of $f(2)$?

A) -2
 B) 3
 C) 5
 D) 19

24. A function a satisfies $a(-2) = 3$ and $a(3) = 8$. A function b satisfies $b(3) = 4$ and $b(7) = -2$. What is the value of $a(b(7))$?



	/	/	
.	.	.	.
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9



25. The function f , shown in the graph above, is defined for $-7 \leq x \leq 7$. For which of the following values of x does $f(x) = 4$?

I. -4
 II. -3
 III. 5

A) III only
 B) I and II only
 C) II and III only
 D) I, II, and III

Reflect

Directions: Take a few minutes to recall what you've learned and what you've been practicing in this chapter. Consider the following questions, jot down your best answer for each one, and then compare your reflections to the expert responses on the following page. Use your level of confidence to determine what to do next.

What are the domain and range of a function?

What is another way to write the function $f(x) = x + 4$?

In the same function, what does x represent? What does $f(x)$ represent?

What will the function look like when graphed?

In a function whose x -value represents time, what does the y -intercept represent?

Expert Responses

What are the domain and range of a function?

The domain of a function indicates the possible x -values and the range of a function indicates the possible y -values. For example, in the function $f(x) = x^2$, the domain is all real numbers because any number can be squared, and the range is any number greater than or equal to 0, because x^2 can't be negative.

What is another way to write the function $f(x) = x + 4$?

When you graph the function on the xy -coordinate plane, you can replace $f(x)$ with y . This function is equivalent to $y = x + 4$.

In the same function, what does x represent? What does $f(x)$ represent?

In this function, x is the input and $f(x)$ is the output.

What will the function look like when graphed?

The slope of the line is 1 and its y -intercept is 4, so it will move from the lower left to the upper right and cross the y -axis at $y = 4$.

In a function whose x -value represents time, what does the y -intercept represent?

The y -intercept represents the initial quantity when $t = 0$. Say a function represents the progress of a machine manufacturing widgets at a rate of 6 widgets per hour. The machine adds the widgets it makes to a growing pile that consisted of 12 widgets when the machine started working. If this function were graphed as a function of time, the y -intercept would be 12—the pile of 12 widgets that were there when the machine started its task.

Next Steps

If you answered most questions correctly in the “How Much Have You Learned?” section, and if your responses to the Reflect questions were similar to those of the SAT expert, then consider functions an area of strength and move on to the next chapter. Come back to this topic periodically to prevent yourself from getting rusty.

If you don't yet feel confident, review those parts of this chapter that you have not yet mastered. All three lessons in this chapter cover question types that are fairly common on the SAT, and it is to your advantage to have a firm grasp on this material, so go back over it until you feel more confident. Then try the questions you missed again. As always, be sure to review the explanations closely. Finally, **go online** (www.kaptest.com/moreonline) for additional practice on the highest yield topics in this chapter.

Answers and Explanations

1. A

Difficulty: Easy

Getting to the Answer: The notation $g(-2)$ is asking for the value of $g(x)$ when x is -2 , so substitute -2 for x and simplify. Don't forget to use the correct order of operations as you work:

$$\begin{aligned} g(-2) &= -2(-2)^2 + 7(-2) - 3 \\ &= -2(4) + (-14) - 3 \\ &= -8 - 14 - 3 \\ &= -25 \end{aligned}$$

(A) is correct.

2. A

Difficulty: Easy

Getting to the Answer: The notation $k(4)$ is equivalent to the output value of the function when 4 is substituted for the input x , and $k(1)$ is the output value of the function when 1 is substituted for the input x . Substitute 4 and 1 into the function, one at a time, and then subtract the results:

$$\begin{aligned} k(4) &= 5(4) + 2 = 20 + 2 = 22 \\ k(1) &= 5(1) + 2 = 5 + 2 = 7 \\ k(4) - k(1) &= 22 - 7 = 15 \end{aligned}$$

Choice (A) is correct. Caution—subtracting 1 from 4 and then substituting 3 into the function will give a different and incorrect result.

3. A

Difficulty: Medium

Getting to the Answer: The notation $g(h(x))$ can be read “ g of h of x .” It means that the output when x is substituted in $h(x)$ becomes the input for $g(x)$. First, use the table on the right to find that $h(3)$ is 0. This is your new input. Now, use the table on the left to find $g(0)$, which is -1 , making (A) the correct answer.

4. D

Difficulty: Medium

Getting to the Answer: Evaluate the numerator and denominator separately:

$$\begin{aligned} p(5) &= 5^2 - 4(5) + 8 = 13 \\ q(p(5)) &= q(13) = 13 - 3 = 10 \\ q(5) &= 5 - 3 = 2 \\ p(q(5)) &= p(2) = 2^2 - 4(2) + 8 = 4 \end{aligned}$$

Next, combine to get $\frac{q(p(5))}{p(q(5))} = \frac{10}{4} = 2.5$. The correct answer is (D).

5. D

Difficulty: Hard

Getting to the Answer: Determine the linear change of the functions relative to the change in n , then extrapolate to get the values of $f(6)$ and $g(6)$. You don't need to determine the actual expressions for the functions. As a shortcut, find the changes per 2 unit increase of n and apply that to the values of the functions when $n = 4$. For $f(n)$, the increase from $n = 2$ to $n = 4$ is $16.2 - 11.6 = 4.6$. Thus, the value of $f(6)$ is $f(4) + 4.6 = 16.2 + 4.6 = 20.8$. The change in $g(n)$ for $n = 2$ to $n = 4$ is $0.5 - 1.5 = -1$. So the value of $g(6)$ is $g(4) + (-1) = 0.5 - 1 = -0.5$. Now, calculate $h(6)$: $h(6) = 2 \times f(6) - g(6) = 2(20.8) - (-0.5) = 41.6 + 0.5 = 42.1$. (D) is correct.

6. C

Difficulty: Easy

Getting to the Answer: The function graphed is the absolute value function, and all of the values in its range (the y -values) are positive. That makes any negative value as an output impossible. Because you're looking for the statement that is *not* true, (C) is correct.

7. D

Difficulty: Easy

Getting to the Answer: To determine the domain, look at the x -values. To determine the range, look at the y -values. For the domain, the graph is continuous (no holes or gaps in the graph) and has arrows on both sides, so the domain is all real numbers. This means you can eliminate (A) and (B). For the range, the function's maximum (the vertex) is located at $(-3, 4)$, which means that the greatest possible y -value of $f(x)$ is 4. The graph is continuous and opens downward, so the range of the function is $y \leq 4$, which is the same as $f(x) \leq 4$, making (D) correct.

8. 8

Difficulty: Medium

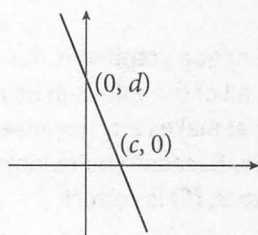
Getting to the Answer: The maximum of $f(x)$ occurs at the point where the y -value is the greatest, which in this case is $(2, 4)$. So, $a = 2$ and $b = 4$. The point with the smallest y -value is $(4, -2)$. Thus, $c = 4$ and $d = -2$. The total of the four values is $2 + 4 + 4 + (-2) = 8$. Grid in 8.

9. B

Difficulty: Medium

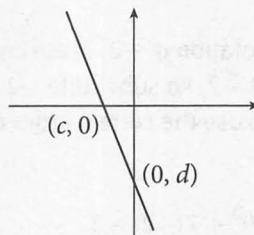
Strategic Advice: Quickly sketching the different possibilities can be helpful.

Getting to the Answer: Because $2c = d$, both the x -intercept, c , and the y -intercept, d , must have the same sign. If both are positive, then d would be greater than c , and the graph of f would look something like this:



This is all you need to do to solve the question. According to the choices, the slope is always the same regardless of the sign of c and d . In other words, if the slope is negative at one point, then it must be negative all the time. Therefore, (B) is correct. On test day, you would move on to the next question without needing to

check what the line looks like when c and d are negative. For the record, if c and d are negative, then d will be less than c , and the graph would look like this:



If you're curious to see the algebra, plug $(0, d)$ and $(c, 0)$ into the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{d - 0}{0 - c} = -\frac{d}{c}$$

The question states that $d = 2c$, so sub in $2c$ for d :

$$-\frac{d}{c} = -\frac{(2c)}{c} = -2$$

Therefore, the slope is -2 , and the answer is indeed (B).

10. D

Difficulty: Easy

Strategic Advice: Save time on questions with roman numerals in the choices by testing the roman numeral that appears most often in the choices first. Here, check statement II first because it appears in three of the four choices.

Getting to the Answer: Plug each of the x -values into the function and see which produces a y -value of -1 . Statement II is $f(0)$. When $x = 0$, the function's y -value is 2. Statement II is not equal to -1 , so any answer choices that include statement II are incorrect. Eliminate (A), (B), and (C). Only (D) is left and is correct. On test day, you would stop here and move on to the next question. For the record, statement I is $f(-4) = 0$ and statement III is $f(3) = -1$. Also note that there are two other places where the function's output is -1 : $f(-3)$ and $f(-1.5)$, neither of which is an answer choice.

11. A

Difficulty: Medium

Getting to the Answer: Thinking about the y -intercept (the starting amount) for the function will reduce the amount of work you need to do. The table indicates that Paulo had 21 votes on day 3, when $t = 3$, not at the start of the contest, when $t = 0$. This means that (C) and (D) are incorrect. To evaluate the remaining answer choices, pick a point, try it in a function, and if it works, then you've found the correct answer. If it doesn't work, then you can confidently select the other answer choice without any further work:

$$\begin{aligned} \text{(A): } 35 &= 2(4)^2 + 3 \\ 35 &= 2(16) + 3 \\ 35 &= 32 + 3 \\ 35 &= 35 \end{aligned}$$

(A) is correct. On test day, you would stop here. For the record, here is the reason **(B)** is incorrect:

$$\begin{aligned} \text{(B): } 35 &= \frac{1}{2}(4)^2 + 3 \\ 35 &= \frac{1}{2}(16) + 3 \\ 35 &= 8 + 3 \\ 35 &\neq 11 \end{aligned}$$

12. C

Difficulty: Medium

Getting to the Answer: Examine the graph and look for trends in the rate of increase and decrease of fuel economy before and after 50 mph. Note the increase below 50 mph (to the left of 50 on the horizontal axis) and the decrease above 50 mph (to the right of 50): the decreasing part of the graph is steeper than the increasing part. In other words, the rate of increase below 50 mph is less than the rate of increase above 50 mph. Choice **(C)** is correct.

13. D

Difficulty: Medium

Getting to the Answer: Compare each answer choice to the graph, eliminating false statements as you go.

(A): Carmel went to the library first, so the library (not the grocery store) is about 5 miles from her home. Eliminate **(A)**.

(B): Carmel traveled 7 miles away from her home (between $t = 0$ minutes and $t = 30$ minutes), but then also traveled 7 miles back (between $t = 45$ minutes and $t = 60$ minutes), so she traveled a total of 14 miles. Eliminate **(B)**.

(C): When Carmel reached the library, she was 5 miles from home; when she reached the grocery store, she was 7 miles from home. This means the grocery store must be $7 - 5 = 2$ miles farther away. Eliminate **(C)**.

(D) must be correct. Carmel is the same distance from home (5 miles) between $t = 15$ minutes and $t = 25$ minutes, so she spent 10 minutes at the library. She is stopped once again (at the grocery store) between $t = 30$ minutes and $t = 45$ minutes, so she spent 15 minutes at the grocery store.

14. C

Difficulty: Medium

Strategic Advice: The fact that the number of visitors each 15 minutes is constant means that the cumulative number of visitors is a linear function.

Getting to the Answer: Because the time between the numbers of cumulative visitors in the table varies, pick an interval that is easy to work with to determine the number of visitors who enter every 15 minutes. Next, use that value to find how many entered by 4:45 p.m. There are six 15-minute periods between 12:30 p.m. and 2:00 p.m. The number of visitors admitted during that time was $600 - 420 = 180$. So, $\frac{180}{6} = 30$ visitors enter every 15 minutes.

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In order to project the cumulative, or total, number of visitors for a specific time, set up a function, $f(v)$. Pick a time that is convenient, such as 2:00 p.m. Since you know that there were 600 visitors by 2:00 p.m. you can write $f(v) = 600 + 30v$, where v is the number of 15-minute periods after 2:00 p.m. The question asks for the cumulative visitors admitted by 4:45 p.m. Thus, v is the number of 15-minute periods between 2:00 and 4:45, which is 11. So, $f(11) = 600 + 30(11) = 930$. **(C)** is correct.

15. B

Difficulty: Medium

Strategic Advice: When modeling a real-life situation with a linear function, the starting point in the description is the y -intercept of the equation and the rate of change is the slope. Eliminate choices as you go; you may find that you are able to answer the question after only one or two steps. Never do more math than necessary to answer the question.

Getting to the Answer: In this question, the agency is reducing the amount of plastic disposed annually, so the slope must be negative. Eliminate (C) and (D) because their positive slopes indicate an *increasing* function.

The starting point, or y -intercept, is the amount of plastic the community is now discarding, or 6.2 million pounds. Unfortunately, this value is the same in (A) and (B), so you will need to find the slope.

The agency wants to eliminate the total amount of plastic in 8 years, so to find the amount of reduction per year, divide 6.2 by 8.

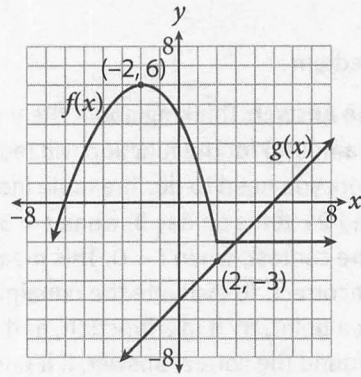
Because $-\frac{6.2}{8}$ does not appear in the choices, and the slopes in the choices do not have decimal points, multiply the fraction by 1 in the form of $\frac{10}{10}$ to get $-\frac{6.2}{8} \times \frac{10}{10} = -\frac{62}{80} = -\frac{31}{40}$. Choice **(B)** is correct.

16. C

Difficulty: Medium

Category: Graphs of Functions

Getting to the Answer: Graphically, the notation $f(-2)$ means the y -value when x is -2 . Pay careful attention to which graph is which. It may help to draw dots on the graph. Find $x = -2$ along the horizontal axis, trace up to the graph of $f(x)$, and draw a dot on the graph. Do the same for $g(2)$, as shown here:



Now, read the y -coordinates from the graph and add: $f(-2)$ is 6 and $g(2)$ is -3 , so $f(-2) + g(2) = 6 + (-3) = 3$, which is **(C)**.

17. 3.5 or 3.50

Difficulty: Medium

Category: Describing Real-Life Situations with Functions

Getting to the Answer: Start by evaluating the function at $x = 25$ and at $x = 20$. Make sure you follow the correct order of operations as you simplify:

$$\begin{aligned} P(25) &= 150(25) - (25)^2 \\ &= 3,750 - 625 \\ &= 3,125 \end{aligned}$$

$$\begin{aligned} P(20) &= 150(20) - (20)^2 \\ &= 3,000 - 400 \\ &= 2,600 \end{aligned}$$

The question asks how much more profit *per unit* the company makes, so find the difference in the amounts of profit and divide by the number of units (150) to get $\frac{3,125 - 2,600}{150} = \frac{525}{150} = \3.50 . Grid in **3.5**.

18. 305**Difficulty:** Easy**Category:** Describing Real-Life Situations with Functions**Getting to the Answer:** At the end of one workday, 10 hours have passed, so evaluate the function at $t = 10$. Make sure you follow the correct order of operations as you simplify:

$$\begin{aligned} C(t) &= -0.0815t^4 + t^3 + 12t \\ C(10) &= -0.0815(10)^4 + 10^3 + 12(10) \\ &= -0.0815(10,000) + 1,000 + 120 \\ &= -815 + 1,000 + 120 \\ &= 305 \end{aligned}$$

Grid in **305**.**19. D****Difficulty:** Easy**Category:** Describing Real-Life Situations with Functions**Getting to the Answer:** The domain of a function includes every possible input value, which is usually represented by x . In this function, instead of x , the input is represented by p , which is the number of seeds germinated by the plants over a given period of time. Because there cannot be a negative number of seeds germinated or a fraction of a seed germinated, the list in **(D)** is the only one that could represent a portion of the function's domain.**20. D****Difficulty:** Hard**Category:** Graphs of Functions**Getting to the Answer:** When working with a composition (also called nested functions), the range of the inner function becomes the domain of the outer function, which in turn produces the range of the composition. In the composition $f(g(x))$, the function $g(x) = \frac{x^2}{2}$ is the inner function. Every value of x , when substituted into this function, will result in a nonnegative value because the result of squaring a number is always a positive number. This means the smallest possible range value of $g(x)$ is 0. If you don't see this relationship, try plugging in various values for x and look for a pattern. Now look at $f(x)$. Substituting large positive values of x in the function will result inlarge negative numbers. Consequently, substituting the smallest value from the range of g , which is 0, results in the largest range value for the composition, which is $3 - 0 = 3$. Therefore, 4 is not in the range of $f(g(x))$. **(D)** is correct.**21. C****Difficulty:** Medium**Category:** Function Notation**Getting to the Answer:** To evaluate a function at a particular value, replace the variable in the function with the value. In this question, replace x in the function definition with $-4x$ to get $g(-4x) = -3(-4x) - 5 = 12x - 5$. **(C)** is correct.Note that you can replace one variable with another. In this question, $-4x$ replaces x . If you chose **(A)**, you likely replaced x in the function definition with -4 . If you chose **(B)**, you may have "lost" a negative sign. Finally, **(D)** results if you multiply each term in the definition by $-4x$ instead of replacing x with $-4x$.**22. D****Difficulty:** Medium**Category:** Function Notation**Getting to the Answer:** To evaluate the value of a function, replace the variable in the definition with the given value. Since the question asks for $t(2)$, and the definition of the t function includes the r function, evaluate $r(2)$ first, then use that result to evaluate $t(2)$:

$$r(2) = 3(2) - 7 = 6 - 7 = -1$$

Because $t(2) = 3(2) + r(2) = 6 - 1 = 5$, **(D)** is correct.

Part 2C
Passport to Advanced Math

23. B

Difficulty: Medium

Category: Function Notation

Getting to the Answer: To evaluate a function for a particular value, substitute the value for the variable in the function. In this case, substitute 3 for x and then use the given value for $f(3)$ to solve for the value of a :

$$f(3) = a(3)^2 + 3(3) + 5$$

$$-4 = 9a + 9 + 5$$

$$-4 = 9a + 14$$

$$-18 = 9a$$

$$-2 = a$$

Next, use the known value of a to find $f(2)$:

$$f(2) = -2(2)^2 + 3(2) + 5$$

$$f(2) = -2(4) + 6 + 5$$

$$f(2) = -8 + 11$$

$$f(2) = 3$$

(B) is correct.

24. 3

Difficulty: Medium

Category: Function Notation

Getting to the Answer: To evaluate a composite function (a set of nested functions), start with the innermost function and work outward. For this question, begin by evaluating $b(7)$. Because $b(7) = -2$, you know that $a(b(7)) = a(-2)$, so next, evaluate $a(-2)$. The question tells you that $a(-2) = 3$. Grid in **3**.

25. C

Difficulty: Easy

Category: Graphs of Functions

Getting to the Answer: The term $f(x)$ is equivalent to the y -value of the function at x . So this question is asking, "Which values of x produce a y -value of 4?" Draw a horizontal line at the point $y = 4$ and identify where that line intercepts the function. The two points of intersection are -3 and 5 . **(C)** is correct.