Answer Key

Section 19-1

1. 38	2. 38	3. 135	4. 9	5. D
6. C				

Section 19-2

1. 6	2. 81	3. 32	4. B	5. D

Section 19-3

1. 48	2. 24	3. 90	4. 32	5. D
6. B				

Section 19-4

1. C	2. C	3. A	4. B

Section 19-5

1. D	2. C	3. A	4. B	5. 14

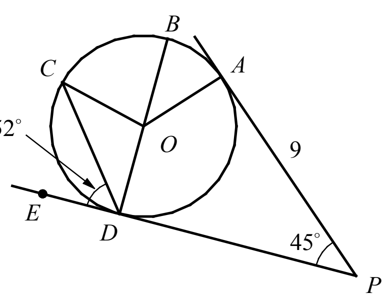
Chapter 19 Practice Test

1. B	2. C	3. A	4. D	5. B
6. D	7. A	8. C	9. $\frac{1}{3}$	10. 20

Answers and Explanations

Section 19-1

1. 38



$\overline{PD} \perp \overline{OD}$ Tangent to a $\odot$ is $\perp$ to radius.
$m\angle ODE = 90$ A right $\angle$ measures 90.
$m\angle ODC = 90 - 52$
$= 38$

2. 38

$OC = OD$ In a $\odot$ all radii are $\cong$.
$m\angle OCD = m\angle ODC$ Isosceles Triangle Theorem
$= 38$

3. 135

If a line is tangent to a circle, the line is $\perp$ to the radius at the point of tangency. Therefore, $m\angle ODP = m\angle OAP = 90$.
The sum of the measures of interior angles of quadrilateral is 360. Therefore,
$m\angle AOD + m\angle ODP + m\angle OAP + m\angle P = 360$.

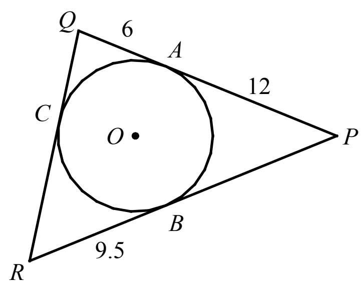
$m\angle AOD + 90 + 90 + 45 = 360$ Substitution
$m\angle AOD + 225 = 360$ Simplify.
$m\angle AOD = 135$

4. 9

Tangents to a circle from the same exterior point are congruent. Therefore,

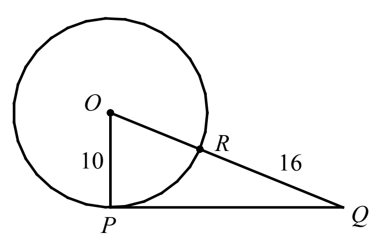
$PD = PA = 9$.

5. D



Since tangents to a circle from the same exterior point are congruent, $QA = QC = 6$, $PA = PB = 12$, and $RB = RC = 9.5$. Therefore,
Perimeter of $\Delta PQR = 2(6 + 12 + 9.5) = 55$

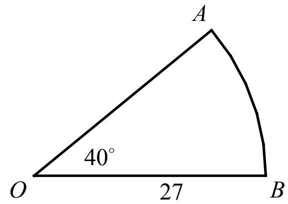
6. C



$OR = OP = 10$ In a $\odot$ all radii are $\cong$.
$OQ = OR + RQ$ Segment Addition Postulate
$= 10 + 16 = 26$
$PQ^2 + OP^2 = OQ^2$ Pythagorean Theorem
$PQ^2 + 10^2 = 26^2$ Substitution
$PQ^2 = 26^2 - 10^2 = 576$
$PQ = \sqrt{576} = 24$

Section 19-2

1. 6



Length of arc $AB = 2\pi r \cdot \frac{m\angle AOB}{360}$

$= 2\pi(27) \cdot \frac{40}{360} = 6\pi$

Thus, $k = 6$.

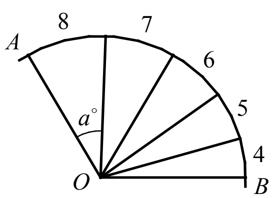
2. 81

Area of sector $OAB = \pi r^2 \cdot \frac{m\angle AOB}{360}$

$= \pi(27)^2 \cdot \frac{40}{360} = 81\pi$

Thus, $n = 81$.

3. 32

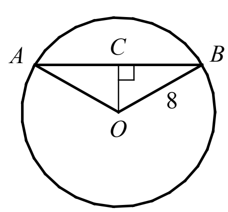


The length of arc $AB = 8 + 7 + 6 + 5 + 4 = 30$
In a circle, the lengths of the arcs are proportional to the degree measures of the corresponding arcs.

Therefore, $\frac{\text{length of arc } AB}{120^\circ} = \frac{8}{a^\circ}$.

$\frac{30}{120} = \frac{8}{a}$	Substitution
$30a = 120 \times 8$	Cross Products
$a = 32$	

4. B



Draw $\overline{OC}$ perpendicular to $\overline{AB}$. Since ΔAOB is an isosceles triangle, $\overline{OC}$ bisects $\angle AOB$.

$m\angle AOC = m\angle BOC = \frac{1}{2} m\angle AOB = \frac{1}{2}(120) = 60$.

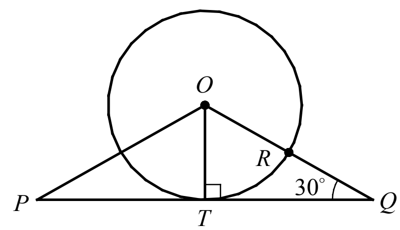
ΔBOC is a 30°-60°-90° triangle.
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

$OC = \frac{1}{2} OB = \frac{1}{2}(8) = 4$

$BC = \sqrt{3} \cdot OC = 4\sqrt{3}$

$AB = 2BC = 2 \times 4\sqrt{3} = 8\sqrt{3}$

5. D



Let T be a point of tangency. Then $\overline{PQ} \perp \overline{OT}$, because a line tangent to a circle is $\perp$ to the radius at the point of tangency.

ΔOQT is a 30°-60°-90° triangle.

$OT = OR = 8$ In a $\odot$ all radii are $\cong$.

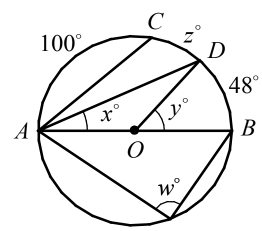
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg. Therefore,

$OQ = 2OT = 2(8) = 16$

$QR = OQ - OR = 16 - 8 = 8$

Section 19-3

1. 48



The measure of a minor arc is the measure of its central angle. Therefore, $y = 48$.

2. 24

The measure of an inscribed angle is half the measure of its intercepted arc.

Therefore, $x = \frac{1}{2}(48) = 24$.

3. 90

An angle inscribed in a semicircle is a right angle. Therefore, $w = 90$.

4. 32

The measure of a semicircle is 180, thus $m\overset{\frown}{ACB} = 180$.
The measure of an arc formed by two adjacent arcs is the sum of the measure of the two arcs, thus

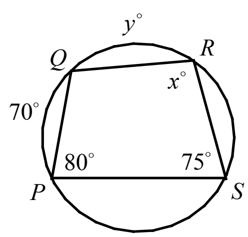
$m\overset{\frown}{ACB} = m\overset{\frown}{AC} + m\overset{\frown}{CD} + m\overset{\frown}{DB}$

$180 = 100 + z + 48$ Substitution

$180 = 148 + z$ Simplify.

$32 = z$

5. D



If a quadrilateral is inscribed in a circle, its opposite angles are supplementary. Therefore, $x + 80 = 180$.
$x = 100$

6. B

The measure of an inscribed angle is half the measure of its intercepted arc. Therefore,

$m\angle RSP = \frac{1}{2}(m\overset{\frown}{PQ} + m\overset{\frown}{QR})$.

$75 = \frac{1}{2}(70 + y)$ Substitution

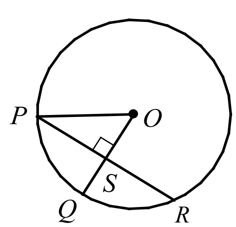
$2 \cdot 75 = 2 \cdot \frac{1}{2}(70 + y)$ Multiply each side by 2.

$150 = 70 + y$ Simplify.

$80 = y$

Section 19-4

1. C



If a diameter is $\perp$ to a chord, it bisects the chord and its arc. Therefore,

$PS = \frac{1}{2}PR = \frac{1}{2}(24) = 12$.

The radius of the circle is 13, thus $OP = OQ = 13$.

Draw $\overline{OP}$.

$OS^2 + PS^2 = OP^2$ Pythagorean Theorem

$OS^2 + 12^2 = 13^2$ Substitution

$OS^2 = 13^2 - 12^2 = 25$

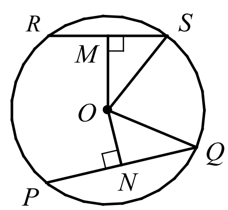
$OS = \sqrt{25} = 5$

$QS = OQ - OS$

$= 13 - 5$

$= 8$

2. C



Draw $\overline{OS}$ and $\overline{OQ}$.

If a diameter is $\perp$ to a chord, it bisects the chord and its arc. Therefore,

$MS = \frac{1}{2}RS = \frac{1}{2}(6) = 3$ and $PQ = 2NQ$.

$OS^2 = MS^2 + OM^2$ Pythagorean Theorem

$OS^2 = 3^2 + 5^2$ Substitution

$OS^2 = 34$

$OS = \sqrt{34}$

$OQ = OS = \sqrt{34}$ In a $\odot$ all radii are $\cong$.

$OQ^2 = ON^2 + NQ^2$ Pythagorean Theorem

$(\sqrt{34})^2 = 4^2 + NQ^2$ Substitution

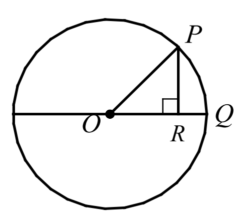
$34 = 16 + NQ^2$

$18 = NQ^2$

$NQ = \sqrt{18} = 3\sqrt{2}$

$PQ = 2NQ = 2(3\sqrt{2}) = 6\sqrt{2}$

3. A



Area of the circle $= \pi r^2 = 9\pi$.

$\Rightarrow\ r^2 = 9\ \Rightarrow\ r = 3$

Therefore, $OP = OQ = 3$.

$OR^2 + PR^2 = OP^2$ Pythagorean Theorem

$OR^2 + (\sqrt{5})^2 = 3^2$ Substitution

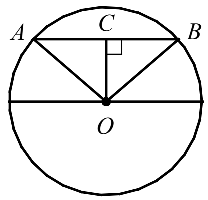
$OR^2 + 5 = 9$ Simplify.

$OR^2 = 9 - 5 = 4$

$OR = \sqrt{4} = 2$

$QR = OQ - OR = 3 - 2 = 1$

4. B



Draw $\overline{OA}$ and $\overline{OB}$. Draw $\overline{OC}$ $\perp$ to $\overline{AB}$. OC is the distance between the chord and the diameter.

$BC = \frac{1}{2}AB = \frac{1}{2}(18) = 9$

$OC^2 + BC^2 = OB^2$ Pythagorean Theorem

$OC^2 + 9^2 = 12^2$ Substitution

$OC^2 = 12^2 - 9^2 = 63$

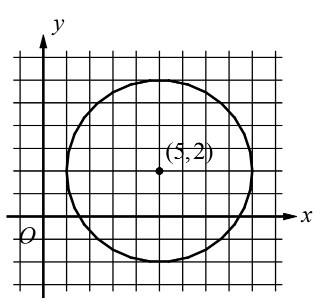
$OC = \sqrt{63}$

$= \sqrt{9} \cdot \sqrt{7}$

$= 3\sqrt{7}$

Section 19-5

1. D



The equation of a circle with center (h,k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

The center of the circle shown above is $(5,2)$ and the radius is 4. Therefore, the equation of the circle is $(x-5)^2 + (y-2)^2 = 4^2$.

2. C

Use the distance formula to find the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $(x_1, y_1) = (-2, 0)$

$= \sqrt{(0-(-2))^2 + (\frac{3}{2} - 0)^2}$ $(x_2, y_2) = (0, \frac{3}{2})$

$= \sqrt{4 + \frac{9}{4}}$ Simplify.

$= \sqrt{\frac{16}{4} + \frac{9}{4}} = \sqrt{\frac{25}{4}}$

Therefore, the equation of the circle is

$(x-(-2))^2 + (y-0)^2 = (\sqrt{\frac{25}{4}})^2$.

Choice C is correct.

3. A

$x^2 + 12x + y^2 - 4y + 15 = 0$

Isolate the constant onto one side.

$x^2 + 12x + y^2 - 4y = -15$

Add $(12 \cdot \frac{1}{2})^2 = 36$ and $(-4 \cdot \frac{1}{2})^2 = 4$ to each side.

$(x^2 + 12x + 36) + (y^2 - 4y + 4) = -15 + 36 + 4$

Complete the square.

$(x+6)^2 + (y-2)^2 = 25$

The center of the circle is $(-6, 2)$ and the radius is $\sqrt{25}$, or 5.

4. B

The center of the circle is the midpoint of the diameter. Use the midpoint formula to find the center of the circle.

$(h,k) = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

$= (\frac{-8+2}{2}, \frac{4+(-6)}{2}) = (-3, -1)$

The radius is half the distance of the diameter. Use the distance formula to find the diameter.

$d = \sqrt{(2-(-8))^2 + (-6-4)^2} = \sqrt{100 + 100}$

$= \sqrt{200} = \sqrt{100} \cdot \sqrt{2} = 10\sqrt{2}$

$r = \frac{1}{2}d = \frac{1}{2}(10\sqrt{2}) = 5\sqrt{2}$

Therefore, the equation of the circle is

$(x-(-3))^2 + (y-(-1))^2 = (5\sqrt{2})^2$, or

$(x+3)^2 + (y+1)^2 = 50$.

5. 14

$x^2 + 2x + y^2 - 4y - 9 = 0$

Isolate the constant onto one side.

$x^2 + 2x + y^2 - 4y = 9$

Add $(2 \cdot \frac{1}{2})^2 = 1$ and $(-4 \cdot \frac{1}{2})^2 = 4$ to each side.

$(x^2 + 2x + 1) + (y^2 - 4y + 4) = 9 + 1 + 4$

Complete the square.

$(x+1)^2 + (y-2)^2 = 14$

The radius of the circle is $\sqrt{14}$.

Area of the circle is $\pi r^2 = \pi(\sqrt{14})^2 = 14\pi$.

Therefore, $k = 14$.

Chapter 19 Practice Test

1. B

An angle inscribed in a semicircle is a right angle. Therefore, $\angle ACB = 90$.

So, ΔABC is a 30°-60°-90° triangle.

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

$AC = \sqrt{3}BC$

$4\sqrt{3} = \sqrt{3}BC$ $\quad AC = 4\sqrt{3}$

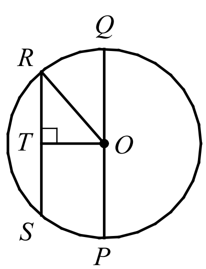
$4 = BC$

$AB = 2BC = 2(4) = 8$

Therefore, the radius of circle O is 4.

Area of circle $O = \pi(4)^2 = 16\pi$

2. C



Draw $\overline{OR}$ and $\overline{OT}$ as shown above. Let the radius of the circle be r, then $OQ = OR = r$.

Since the ratio of RS to QP is 3 to 4, the ratio of RT to OQ is also 3 to 4.

Therefore, $RT = \frac{3}{4}OQ = \frac{3}{4}r$.

OT is the distance between the chord and the diameter, which is given as $2\sqrt{7}$.

$OR^2 = RT^2 + OT^2$ Pythagorean Theorem

$r^2 = (\frac{3}{4}r)^2 + (2\sqrt{7})^2$ Substitution

$r^2 = \frac{9}{16}r^2 + 28$ Simplify.

$r^2 - \frac{9}{16}r^2 = 28$

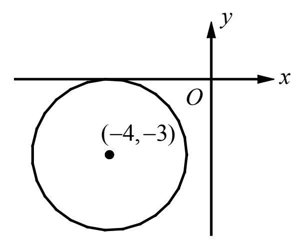
$\frac{7}{16}r^2 = 28$

$\frac{16}{7} \cdot \frac{7}{16}r^2 = \frac{16}{7} \cdot 28$

$r^2 = 64$

$r = \sqrt{64} = 8$

3. A

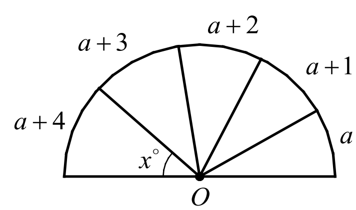


If the center of the circle is $(-4, -3)$ and the circle is tangent to the x-axis, the radius is 3.

The equation is $(x - (-4))^2 + (y - (-3))^2 = 3^2$,

or $(x+4)^2 + (y+3)^2 = 9$.

4. D



The arc length of the semicircle is

$(a+4) + (a+3) + (a+2) + (a+1) + a = 5a + 10$.

In a circle, the lengths of the arcs are proportional to the degree measures of the corresponding arcs.

Therefore, $\frac{\text{arc length of semicircle}}{180^\circ} = \frac{a+4}{x^\circ}$.

$\frac{5a+10}{180} = \frac{a+4}{42}$ Substitution

$42(5a+10) = 180(a+4)$ Cross Products

$210a + 420 = 180a + 720$

$30a = 300$

$a = 10$

5. B

Length of arc $AB = 2\pi r \cdot \frac{m\angle AOB}{360}$

$= 2\pi r \cdot \frac{36}{360} = \frac{\pi r}{5}$

Since the length of the arc is given as π,

$\frac{\pi r}{5} = \pi$. Solving the equation for r gives $r = 5$.

Area of sector $AOB = \pi r^2 \cdot \frac{m\angle AOB}{360}$

$= \pi(5)^2 \cdot \frac{36}{360} = \frac{5}{2}\pi$

6. D

$x^2 - 4x + y^2 - 6x - 17 = 0$

$x^2 - 4x + y^2 - 6x = 17$

To complete the square, add $(-4 \cdot \frac{1}{2})^2 = 4$ and

$(-6 \cdot \frac{1}{2})^2 = 9$ to each side.

$x^2 - 4x + 4 + y^2 - 6x + 9 = 17 + 4 + 9$

$(x-2)^2 + (y-3)^2 = 30$

The radius of the circle is $\sqrt{30}$, the area of the circle is $\pi(\sqrt{30})^2 = 30\pi$

7. A

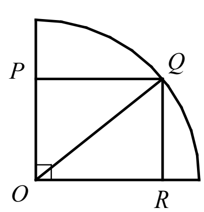
If the diameter of the circle is 8 units, the radius of the circle is 4 units. Since the radius of the circle is 4 units, the y- coordinate of the center has to be 4 units above or below $y = 2$.

The y- coordinate of the center has to be either 6 or -2. Among the answer choices, only choice A has -2 as the y- coordinate.

No other answer choice has 6 or -2 as the y- coordinate of the center.

Choice A is correct.

8. C



Draw $\overline{OQ}$. Since $\overline{OQ}$ is a radius, $OQ = 9$.

$OP^2 + PQ^2 = OQ^2$ Pythagorean Theorem

$OP^2 + 7^2 = 9^2$ Substitution

$OP^2 = 9^2 - 7^2 = 32$

$OP = \sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

Area of rectangle $OPQR = OP \times PQ$

$= 4\sqrt{2} \times 7 = 28\sqrt{2}$

9. $\frac{1}{3}$

Area of sector $AOB = \pi r^2 \cdot \frac{m\angle AOB}{360}$

The area of a sector is the fractional part of the area of a circle. The area of a sector formed by $\frac{2\pi}{3}$ radians of arc is $\frac{2\pi/3}{2\pi}$, or $\frac{1}{3}$, of the area of the circle.

10. 20

The distance the wheel travels in 1 minute is equal to the product of the circumference of the wheel and the number of revolutions per minute.

The distance the wheel travels in 1 minute

$= 2\pi r \times$ the number of revolutions per minute

$= 2\pi(2.2 \text{ ft}) \times 400 = 1{,}760\pi$ ft

Total distance traveled in 1 hour

$= 1{,}760\pi \text{ ft} \times 60 = 105{,}600\pi$ ft

$= 105{,}600\pi \text{ ft} \times \frac{1 \text{ mile}}{5{,}280 \text{ ft}} = 20\pi$ miles

Thus, $k = 20$.