

# CHAPTER 17

## Triangles

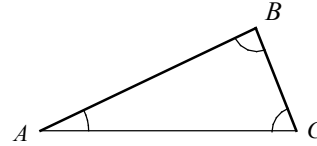
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### 17-1. Angles of a Triangle

#### Angle Sum Theorem

The angle sum of a triangle is  $180^\circ$ .

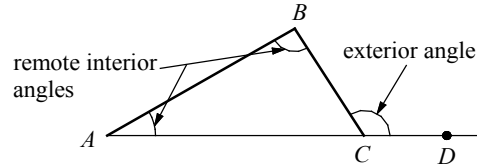
$$m\angle A + m\angle B + m\angle C = 180^\circ$$



#### Exterior Angle Theorem

The measure of an **exterior angle** of a triangle is equal to the sum of the measures of the two remote interior angles.

$$m\angle BCD = m\angle A + m\angle B$$

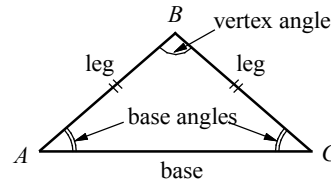


#### Isosceles Triangle Theorem

If two sides of a triangle are congruent, the angles opposite of those sides are congruent.

If  $AB = BC$ , then  $m\angle C = m\angle A$ .

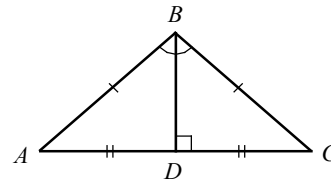
The converse is also true.



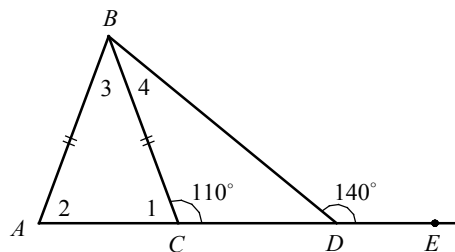
#### Isosceles Triangle Theorem - Corollary

If a line bisects the vertex angle of an isosceles triangle, the line is the perpendicular bisector of the base.

If  $AB = BC$  and  $m\angle ABD = m\angle CBD$ ,  
then  $\overline{BD} \perp \overline{AC}$  and  $AD = CD$ .



Example 1 □ a. In  $\triangle ABC$  shown below,  $AB = BC$ ,  $m\angle BCD = 110$  and  $m\angle BDE = 140$ .  
Find  $m\angle 1$ ,  $m\angle 2$ ,  $m\angle 3$ , and  $m\angle 4$ .

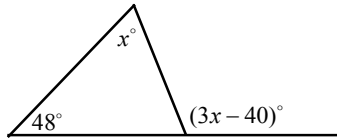


Solution □  $m\angle 1 + m\angle BCD = 180$   
 $m\angle 1 + 110 = 180$   
 $m\angle 1 = 180 - 110 = 70$   
 $m\angle 2 = m\angle 1 = 70$   
 $m\angle 3 + m\angle 2 = 110$   
 $m\angle 3 + 70 = 110$   
 $m\angle 3 = 40$   
 $m\angle 4 + 110 = 140$   
 $m\angle 4 = 30$

Straight angle equals  $180^\circ$ .  
 Substitution  
 Subtraction  
 Isosceles Triangle Theorem  
 Exterior Angle Theorem  
 Substitution  
 Subtraction  
 Exterior Angle Theorem  
 Subtraction

## Exercises - Angles of a Triangle

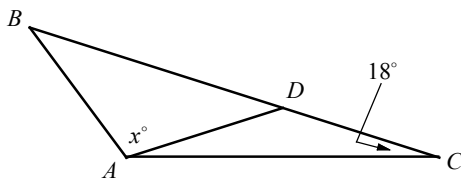
1



In the triangle above, what is the value of  $x$ ?

- A) 44
- B) 48
- C) 52
- D) 56

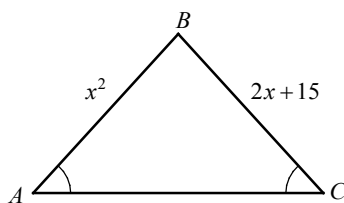
2



In  $\triangle ABC$  above, if  $AB = AD = DC$ , what is the value of  $x$ ?

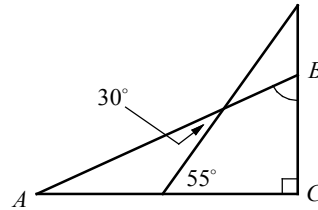
- A) 92
- B) 96
- C) 102
- D) 108

3



In  $\triangle ABC$  above,  $m\angle A = m\angle C$ . If  $x > 0$ , what is the value of  $x$ ?

4

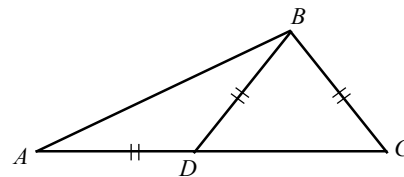


Note: Figure not drawn to scale.

In the figure above,  $AC \perp BC$ . What is the measure of  $\angle ABC$ ?

- A) 50
- B) 55
- C) 60
- D) 65

5



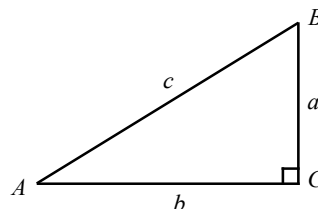
In the figure above,  $AD = BD = BC$ . If  $m\angle A = 26$ , what is the measure of  $m\angle DBC$ ?

- A) 68
- B) 72
- C) 76
- D) 82

17-2. Pythagorean Theorem and Special Right Triangles

A triangle with a right angle is called a **right triangle**. The side opposite to the right angle is called the **hypotenuse** and the other two sides are called **legs**.

In a right triangle the acute angles are complementary. In triangle shown at right,  $m\angle A + m\angle B = 90$ .



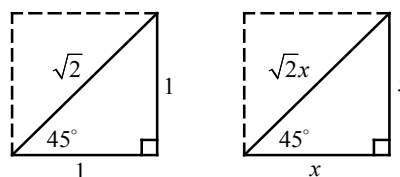
The **Pythagorean theorem** states that in a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.

In right triangle  $ABC$  at the right,  $a^2 + b^2 = c^2$ .

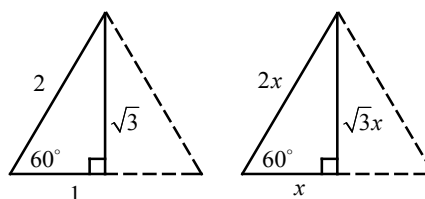
The converse is also true.

The Pythagorean theorem can be used to determine the ratios of the lengths of the sides of two special right triangles.

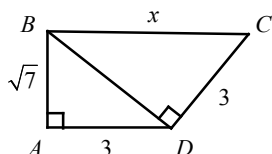
In a  $45^\circ-45^\circ-90^\circ$  triangle, the hypotenuse is  $\sqrt{2}$  times as long as a leg. An isosceles right triangle is also called a  $45^\circ-45^\circ-90^\circ$  triangle.



In a  $30^\circ-60^\circ-90^\circ$  triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is  $\sqrt{3}$  times as long as the shorter leg.



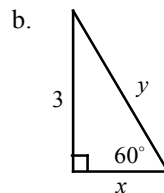
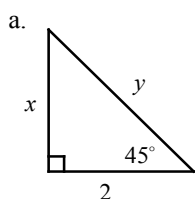
Example 1 □ In the figure below, find the value of  $x$ .



Solution □  $BD^2 = (\sqrt{7})^2 + 3^2 = 16$   
 $x^2 = BD^2 + 3^2$   
 $x^2 = 16 + 9 = 25$   
 $x = \sqrt{25} = 5$

Pythagorean Theorem  
 Pythagorean Theorem  
 Substitution

Example 2 □ In the figures below, find the values of  $x$  and  $y$ .

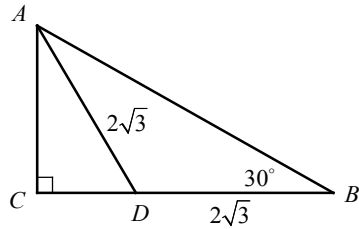


Solution □ a. Since a  $45^\circ-45^\circ-90^\circ$  triangle is an isosceles right triangle,  $x = 2$ .  
 In a  $45^\circ-45^\circ-90^\circ$  triangle, hypotenuse  $= \sqrt{2} \cdot \text{leg} \Rightarrow y = 2\sqrt{2}$

b. In a  $30^\circ-60^\circ-90^\circ$  triangle, longer leg  $= \sqrt{3} \cdot \text{shorter leg} \Rightarrow 3 = \sqrt{3}x \Rightarrow x = \frac{3}{\sqrt{3}} = \sqrt{3}$   
 hypotenuse  $= 2 \cdot \text{shorter leg} \Rightarrow y = 2\sqrt{3}$

## Exercises - Pythagorean Theorem and Special Right Triangles

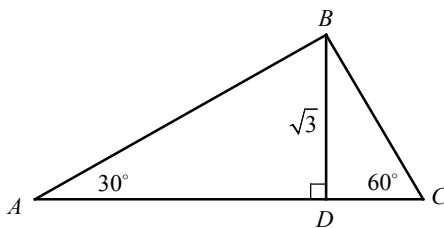
1



In the figure above, if  $AD = BD = 2\sqrt{3}$ , what is the length of  $AB$ ?

- A)  $4\sqrt{3}$
- B)  $3\sqrt{6}$
- C) 6
- D)  $6\sqrt{2}$

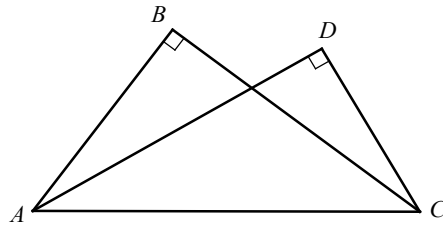
2



In  $\triangle ABC$  above,  $BD = \sqrt{3}$ . What is the perimeter of  $\triangle ABC$ ?

- A)  $2\sqrt{2} + 6$
- B)  $2\sqrt{3} + 6$
- C)  $2\sqrt{6} + 6$
- D)  $3\sqrt{2} + 6$

3

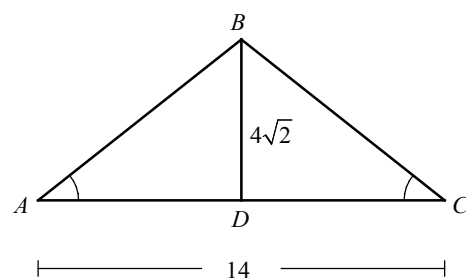


Note: Figure not drawn to scale.

In the figure above,  $AB = 6$ ,  $BC = 8$ , and  $CD = 5$ . What is the length of  $AD$ ?

- A)  $4\sqrt{3}$
- B)  $5\sqrt{2}$
- C)  $5\sqrt{3}$
- D)  $6\sqrt{2}$

4



Note: Figure not drawn to scale.

In the figure above,  $\angle A \cong \angle C$  and  $\overline{BD}$  bisects  $\overline{AC}$ . What is the perimeter of  $\triangle ABC$ ?

- A) 32
- B) 36
- C)  $14 + 10\sqrt{2}$
- D)  $14 + 12\sqrt{2}$

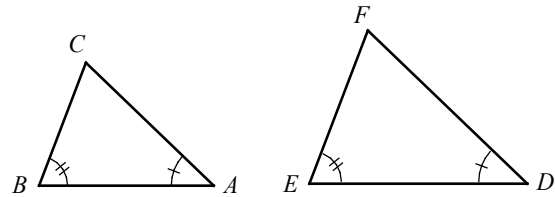
17-3. Similar Triangles and Proportional Parts

**AA Similarity Postulate**

If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.

If two triangles are similar, their corresponding angles are congruent and their corresponding sides are proportional.

If two triangles are similar, their perimeters are proportional to the measures of the corresponding sides.



If  $\angle A \cong \angle D$  and  $\angle B \cong \angle E$ , then  $\angle C \cong \angle F$ .

Therefore  $\triangle ABC \sim \triangle DEF$ , and

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF}$$

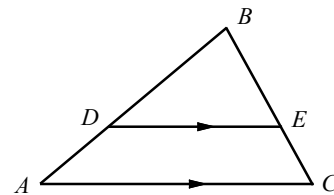
**Triangle Proportionality Theorem**

If a line parallel to one side of a triangle intersects the other two sides, it divides those sides proportionally.

In  $\triangle ABC$ , if  $\overline{AC} \parallel \overline{DE}$  then  $\triangle ABC \sim \triangle DBE$  by AA Similarity.

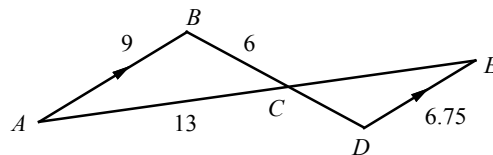
It follows that  $\frac{AB}{DB} = \frac{CB}{EB} = \frac{AC}{DE}$ . Also  $\frac{BD}{DA} = \frac{BE}{EC}$ ,  $\frac{BA}{DA} = \frac{BC}{EC}$ ,

$$\frac{BD}{DE} = \frac{BA}{AC}, \text{ and } \frac{BE}{DE} = \frac{BC}{AC}.$$



If  $D$  and  $E$  are the midpoints of  $\overline{AB}$  and  $\overline{BC}$ ,  $\overline{AC} \parallel \overline{DE}$  and  $DE = \frac{1}{2} AC$ .

Example 1 □ In the figure below,  $\overline{AB} \parallel \overline{DE}$ . Find  $CD$  and  $CE$ .



Solution □  $m\angle A = m\angle E$  (Alternate Interior  $\angle$ s are  $\cong$ .)  
 $m\angle BCA = m\angle DCE$  (Vertical  $\angle$ s are  $\cong$ .)  
 $\triangle ABC \sim \triangle EDC$  (AA Similarity)

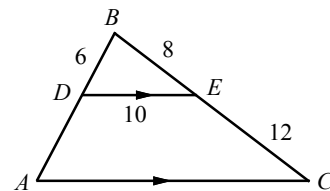
$$\text{Therefore } \frac{AB}{ED} = \frac{BC}{DC} \Rightarrow \frac{9}{6.75} = \frac{6}{DC} \Rightarrow DC = 4.5$$

$$\frac{AB}{ED} = \frac{AC}{EC} \Rightarrow \frac{9}{6.75} = \frac{13}{EC} \Rightarrow EC = 9.75$$

Example 2 □ In the figure at right,  $\overline{AC} \parallel \overline{DE}$ . Find the length of  $\overline{DA}$  and  $\overline{AC}$ .

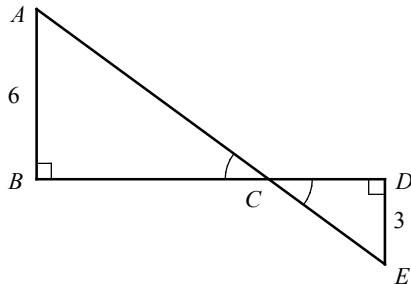
$$\text{Solution } \square \frac{BD}{DA} = \frac{BE}{EC} \Rightarrow \frac{6}{DA} = \frac{8}{12} \Rightarrow DA = 9$$

$$\frac{BD}{DE} = \frac{BA}{AC} \Rightarrow \frac{6}{10} = \frac{6+9}{AC} \Rightarrow AC = 25$$



## Exercises - Similar Triangles and Proportional Parts

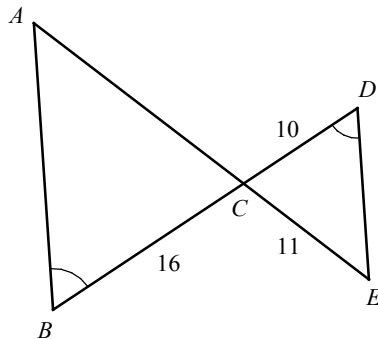
1



In the figure above, if  $AB = 6$ ,  $DE = 3$ , and  $BD = 12$ , what is the length of  $AE$ ?

- A) 12
- B)  $9\sqrt{2}$
- C)  $8\sqrt{3}$
- D) 15

2

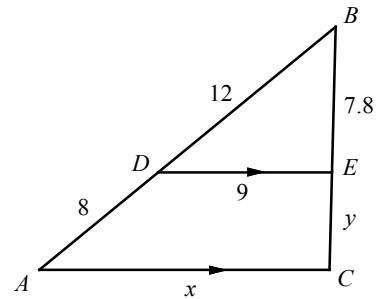


Note: Figure not drawn to scale.

In the figure above,  $\angle B \cong \angle D$ . If  $BC = 16$ ,  $CD = 10$ , and  $CE = 11$ , what is the length of  $AE$ ?

- A) 16.8
- B) 17.2
- C) 17.6
- D) 18.4

Questions 3 and 4 refer to the following information.



In the figure above,  $\overline{DE} \parallel \overline{AC}$ .

3

What is the value of  $x$ ?

- A) 12.5
- B) 15
- C) 16.5
- D) 18

4

What is the value of  $y$ ?

- A) 5.2
- B) 5.6
- C) 6.0
- D) 6.4

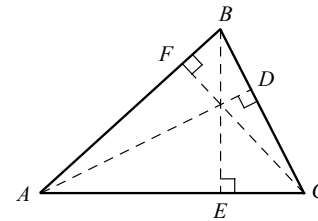
**17-4. Area of a Triangle**

The **area**  $A$  of a triangle equals half the product of a base and the height to that base.

$$A = \frac{1}{2}b \cdot h$$

Area of equilateral triangle with side length of  $a$ .

$$A = \frac{1}{2}(a)\left(\frac{\sqrt{3}}{2}a\right) = \frac{\sqrt{3}}{4}a^2$$



Any side of a triangle can be used as a base. The height that corresponds to the base is the perpendicular line segment from the opposite vertex to the base. The area of  $\triangle ABC$  at the right

can be written in 3 different ways:  $\text{area of } \triangle ABC = \frac{1}{2}BC \cdot AD = \frac{1}{2}AC \cdot BE = \frac{1}{2}AB \cdot CF$ .

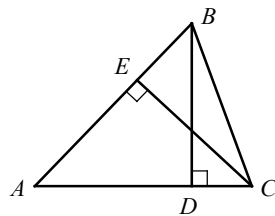
The **perimeter**  $P$  of a triangle is the sum of the lengths of all three sides.

$$P = AB + BC + CA$$

**Ratios of Areas of Two Triangles**

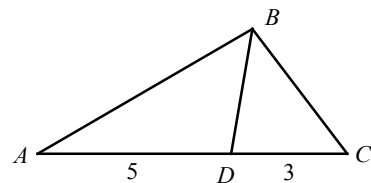
1. If two triangles are similar with corresponding sides in a ratio of  $a : b$ , then the ratio of their areas equals  $a^2 : b^2$ .
2. If two triangles have equal heights, then the ratio of their areas equals the ratio of their bases.
3. If two triangles have equal bases, then the ratio of their areas equals the ratio of their heights.

Example 1 □ In the figure below, if  $AC = 6$ ,  $BD = 4$ , and  $AB = 8$ , what is the length of  $CE$ ?



Solution □ Area of  $\triangle ABC = \frac{1}{2}AC \cdot BD = \frac{1}{2}AB \cdot CE$ .  
 $\Rightarrow \frac{1}{2}(6)(4) = \frac{1}{2}(8)(CE) \Rightarrow CE = 3$

Example 2 □ In the figure below,  $AD = 5$  and  $DC = 3$ . Find the ratio of the area of  $\triangle ABD$  to the area of  $\triangle CBD$ .

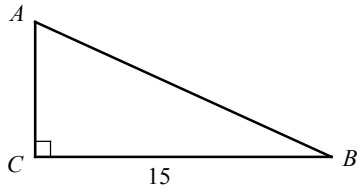


Solution □ The two triangles have the same height, so the ratio of the areas of the two triangles is equal to the ratio of their bases.

$$\frac{\text{area of } \triangle ABD}{\text{area of } \triangle CBD} = \frac{AD}{CD} = \frac{5}{3}$$

## Exercises - Area of a Triangle

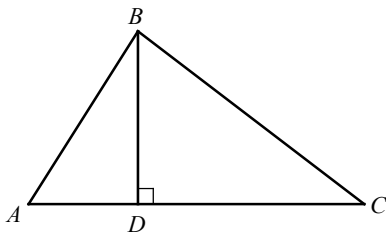
1



In the figure above, the area of right triangle  $ABC$  is 60. What is the perimeter of  $\triangle ABC$ ?

- A) 34
- B) 36
- C) 38
- D) 40

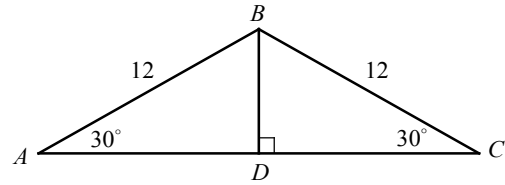
2



In triangle  $ABC$  above, if  $BD$  was increased by 50 percent and  $AC$  was reduced by 50 percent, how would the area of  $\triangle ABC$  change?

- A) The area of  $\triangle ABC$  would be decreased by 25 percent.
- B) The area of  $\triangle ABC$  would be increased by 25 percent.
- C) The area of  $\triangle ABC$  would not change.
- D) The area of  $\triangle ABC$  would be decreased by 50 percent.

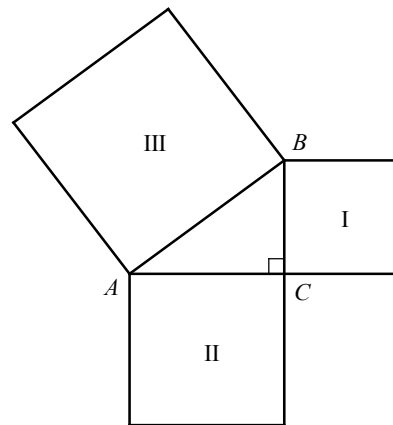
3



In the figure above, what is the area of  $\triangle ABC$ ?

- A)  $24\sqrt{3}$
- B)  $30\sqrt{3}$
- C)  $36\sqrt{3}$
- D)  $48\sqrt{3}$

4



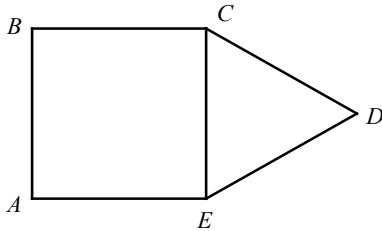
The figure above shows right triangle  $\triangle ABC$  and three squares. If the area of square region I is 80 square inches and the area of square region II is 150 square inches, which of the following is true about the area of square region III?

- A) Less than 230 square inches.
- B) More than 230 square inches.
- C) Equal to 230 square inches.
- D) It cannot be determined from the information given.



## Chapter 17 Practice Test

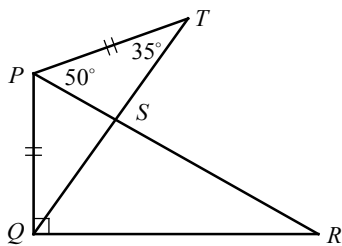
**1**



In the figure above,  $CDE$  is an equilateral triangle and  $ABCD$  is a square with an area of  $4x^2$ . What is the area of triangle  $CDE$  in terms of  $x$ ?

- A)  $\frac{\sqrt{3}}{2}x^2$
- B)  $\sqrt{3}x^2$
- C)  $\frac{3\sqrt{3}}{2}x^2$
- D)  $2\sqrt{3}x^2$

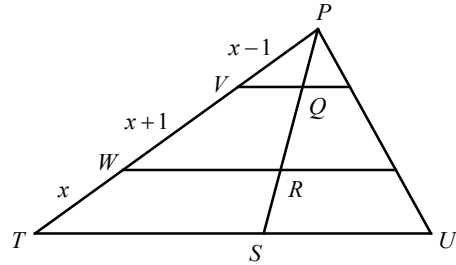
**2**



In the figure above,  $\overline{PQ} \perp \overline{QR}$  and  $\overline{PQ} \cong \overline{PT}$ . What is the measure of  $\angle R$ ?

- A) 30
- B) 35
- C) 40
- D) 45

**3**



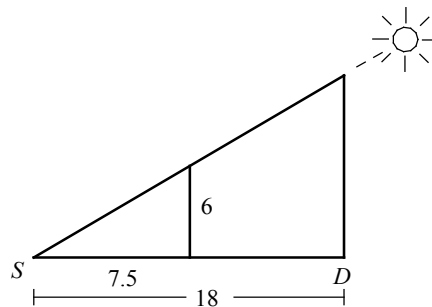
Note: Figure not drawn to scale.

In the figure above,  $\overline{VQ} \parallel \overline{WR} \parallel \overline{TS}$ .

If  $PS = 15$ , what is the length of  $\overline{RS}$ ?

- A) 4.5
- B) 5
- C) 6
- D) 6.5

**4**

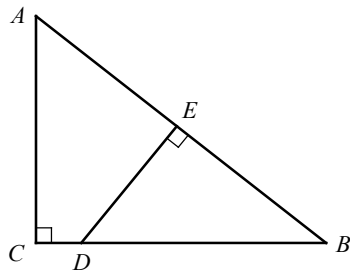


Note: Figure not drawn to scale.

A person 6 feet tall stands so that the ends of his shadow and the shadow of the pole coincide. The length of the person's shadow was measured 7.5 feet and the length of the pole's shadow,  $SD$ , was measured 18 feet. How tall is the pole?

- A) 12.8
- B) 13.6
- C) 14.4
- D) 15.2

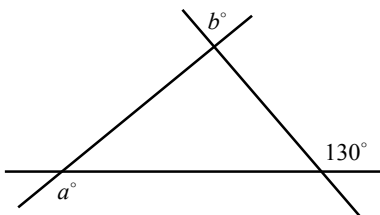
5



In the figure above,  $\triangle ABC$  and  $\triangle DBE$  are right triangles. If  $AC = 12$ ,  $BC = 15$ , and  $DE = 8$ , what is the length of  $BE$ ?

- A) 8.5
- B) 9
- C) 9.5
- D) 10

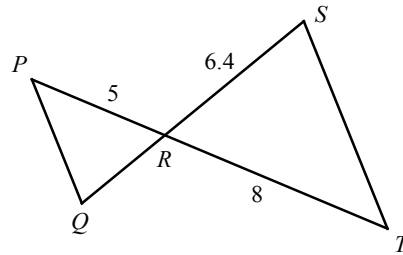
6



In the figure above, what is the value of  $a - b$ ?

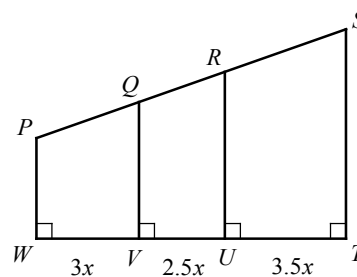
- A) 50
- B) 55
- C) 60
- D) 65

7



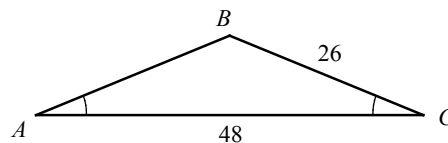
In the figure above,  $\overline{PQ} \parallel \overline{ST}$  and segment  $PT$  intersects segment  $QS$  at  $R$ . What is the length of segment  $QS$ ?

8



In the figure above, if  $PS = 162$ , what is the length of segment  $QR$ ?

9



In the figure above, what is the area of the isosceles triangle  $ABC$ ?

**Answer Key**

Section 17-1

1. A    2. D    3. 5    4. D    5. C

Section 17-2

1. C    2. B    3. C    4. A

Section 17-3

1. D    2. C    3. B    4. A

Section 17-4

1. D    2. A    3. C    4. C

Chapter 17 Practice Test

1. B    2. A    3. B    4. C    5. D  
6. A    7. 10.4    8. 45    9. 240

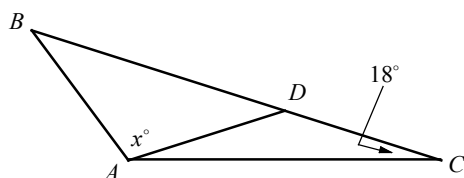
**Answers and Explanations**

**Section 17-1**

1. A

$3x - 40 = x + 48$	Exterior Angle Theorem
$3x - 40 - x = x + 48 - x$	Subtract $x$ from each side.
$2x - 40 = 48$	Simplify.
$2x = 88$	Add 40 to each side.
$x = 44$	

2. D



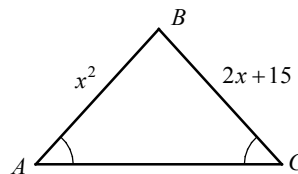
$AD = DC$	Given
$m\angle DAC = m\angle DCA = 18$	Isosceles $\Delta$ Theorem
$m\angle BDA$	Exterior $\angle$ Theorem
$= m\angle DCA + m\angle DAC$	
$m\angle BDA = 18 + 18$	$m\angle DAC = m\angle DCA = 18$
$m\angle BDA = 36$	Simplify.
$AB = AD$	Given
$m\angle DBA = m\angle BDA = 36$	Isosceles $\Delta$ Theorem

In triangle  $ABD$ , the angle sum is 180.

Thus,  $x + 36 + 36 = 180$ .

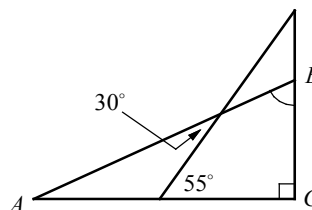
Solving the equation for  $x$  gives  $x = 108$ .

3. 5



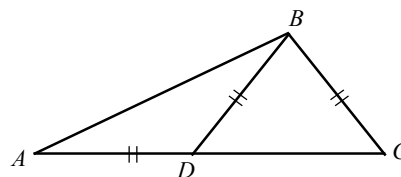
$m\angle A = m\angle C$	Given
$AB = BC$	Isosceles $\Delta$ Theorem
$x^2 = 2x + 15$	Substitution
$x^2 - 2x - 15 = 0$	Make one side 0.
$(x + 3)(x - 5) = 0$	Factor.
$x + 3 = 0$ or $x - 5 = 0$	Zero Product Property
$x = -3$ or $x = 5$	
Since $x > 0$ , the value of $x$ is 5.	

4. D



$m\angle A + 30 = 55$	Exterior Angle Theorem
$m\angle A = 25$	
$m\angle A + m\angle B = 90$	The acute $\angle$ s of a right $\Delta$ are complementary.
$25 + m\angle B = 90$	$m\angle A = 25$ .
$m\angle B = 65$	

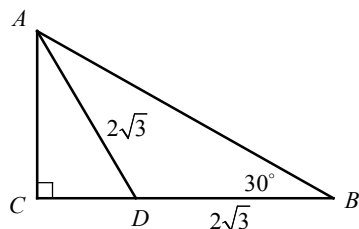
5. C



$AD = BD$	Given
$m\angle ABD = m\angle A$	Isosceles $\Delta$ Theorem
$m\angle A = 26$	Given
$m\angle ABD = 26$	$m\angle A = 26$
$m\angle BDC$	Exterior $\angle$ Theorem
$= m\angle A + m\angle ABD$	
$m\angle BDC = 26 + 26 = 52$	$m\angle A = m\angle ABD = 26$
$BD = BC$	Given
$m\angle C = m\angle BDC$	Isosceles $\Delta$ Theorem
$m\angle C = 52$	$m\angle BDC = 52$
$m\angle C + m\angle BDC + m\angle DBC = 180$	Angle Sum Theorem
$52 + 52 + m\angle DBC = 180$	$m\angle C = m\angle BDC = 52$
$m\angle DBC = 76$	

## Section 17-2

1. C



$AD = BD$  Given  
 $m\angle BAD = m\angle B = 30$  Isosceles  $\Delta$  Theorem  
 $m\angle ADC = m\angle BAD + m\angle B$  Exterior  $\angle$  Theorem  
 $m\angle ADC = 30 + 30 = 60$   $m\angle BAD = m\angle B = 30$   
 $\Delta ADC$  is a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle.

In a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, the hypotenuse is twice as long as the shorter leg. Therefore,

$$AD = 2CD$$

$$2\sqrt{3} = 2CD$$

$$\sqrt{3} = CD.$$

$$BC = BD + CD = 2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$$

Triangle  $ABC$  is also a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle.

In a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, the longer leg is  $\sqrt{3}$  times as long as the shorter leg. Therefore,

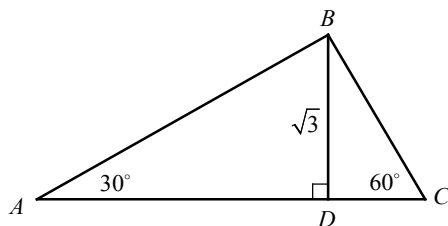
$$BC = \sqrt{3}AC$$

$$3\sqrt{3} = \sqrt{3}AC$$

$$3 = AC.$$

$$AB = 2AC = 2 \times 3 = 6$$

2. B



In the figure above,  $\Delta ABD$  and  $\Delta BCD$  are  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles.

In a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is  $\sqrt{3}$  times as long as the shorter leg. In  $\Delta ABD$ ,

$$AB = 2BD = 2\sqrt{3}$$

$$AD = \sqrt{3}BD = \sqrt{3} \cdot \sqrt{3} = 3.$$

In  $\Delta BCD$ ,

$$BD = \sqrt{3}CD$$

$$\sqrt{3} = \sqrt{3}CD$$

$$1 = CD$$

$$BC = 2CD = 2 \cdot 1 = 2$$

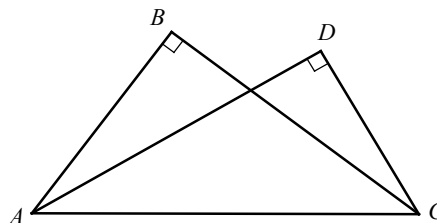
perimeter of  $\Delta ABC$

$$= AB + BC + AC$$

$$= 2\sqrt{3} + 2 + (3+1)$$

$$= 2\sqrt{3} + 6$$

3. C



Note: Figure not drawn to scale.

$$AC^2 = AB^2 + BC^2 \quad \text{Pythagorean Theorem}$$

$$AC^2 = 6^2 + 8^2 = 100$$

$$AC^2 = AD^2 + CD^2 \quad \text{Pythagorean Theorem}$$

$$100 = AD^2 + 5^2 \quad AC^2 = 100, CD = 5$$

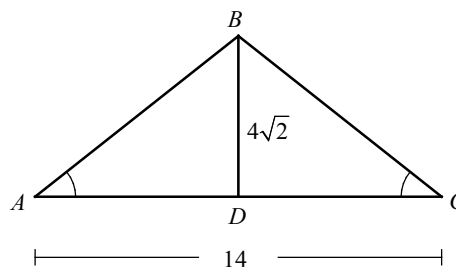
$$100 - 25 = AD^2$$

$$75 = AD^2$$

$$\sqrt{75} = AD$$

$$5\sqrt{3} = AD$$

4. A



Note: Figure not drawn to scale.

$$AD = CD = 7 \quad \text{Definition of segment bisector}$$

$$AB^2 = BD^2 + AD^2 \quad \text{Pythagorean Theorem}$$

$$AB^2 = (4\sqrt{2})^2 + 7^2 \quad \text{Substitution}$$

$$= 32 + 49 = 81$$

$$AB = \sqrt{81} = 9$$

Square root both sides.

$$AB = BC$$

Isosceles Triangle Theorem

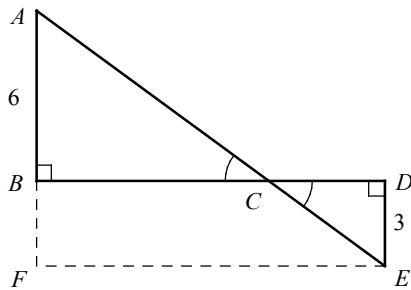
Perimeter of  $\Delta ABC$

$$= AB + BC + AC$$

$$= 9 + 9 + 14 = 32$$

**Section 17-3**

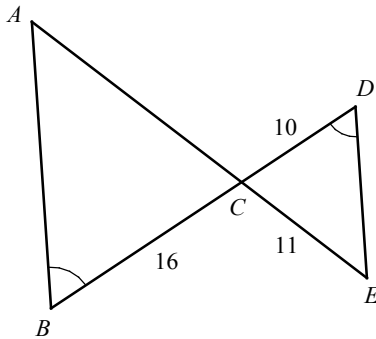
1. D



Draw  $\overline{EF}$ , which is parallel and congruent to  $\overline{BD}$ . Extend  $\overline{AB}$  to point  $F$ . Since  $\overline{EF} \parallel \overline{BD}$ ,  $\angle F$  is a right angle.

$$\begin{aligned} BD &= EF = 12 \text{ and } DE = BF = 3 \\ AF &= AB + BF = 6 + 3 = 9 \\ AE^2 &= AF^2 + EF^2 && \text{Pythagorean Theorem} \\ &= 9^2 + 12^2 \\ &= 225 \\ AE &= \sqrt{225} = 15 \end{aligned}$$

2. C



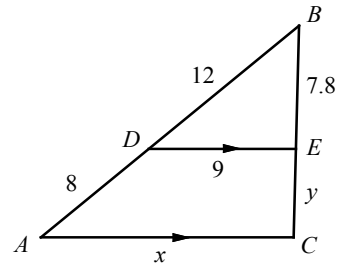
Note: Figure not drawn to scale.

$$\begin{aligned} \angle B &\cong \angle D && \text{Given} \\ \angle ACB &\cong \angle ECD && \text{Vertical } \angle s \text{ are } \cong. \\ \triangle ACB &\sim \triangle ECD && \text{AA similarity} \end{aligned}$$

If two triangles are similar, their corresponding sides are proportional.

$$\begin{aligned} \frac{BC}{DC} &= \frac{AC}{EC} \\ \frac{16}{10} &= \frac{AC}{11} \\ 10AC &= 16 \times 11 \\ AC &= 17.6 \end{aligned}$$

3. B



$$\begin{aligned} \frac{BD}{DE} &= \frac{BA}{AC} \Rightarrow \frac{12}{9} = \frac{20}{x} \Rightarrow 12x = 9 \cdot 20 \\ &\Rightarrow x = 15 \end{aligned}$$

4. A

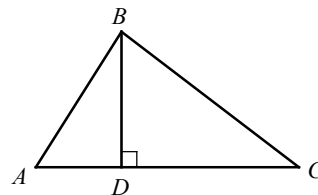
$$\begin{aligned} \frac{BD}{DA} &= \frac{BE}{EC} \Rightarrow \frac{12}{8} = \frac{7.8}{y} \Rightarrow 12y = 8 \times 7.8 \\ &\Rightarrow y = 5.2 \end{aligned}$$

**Section 17-4**

1. D

$$\begin{aligned} \text{Area of triangle } ABC &= \frac{1}{2} BC \cdot AC \\ &= \frac{1}{2} (15) AC = 60 \\ &\Rightarrow 7.5 AC = 60 \Rightarrow AC = 8 \\ AB^2 &= AC^2 + BC^2 && \text{Pythagorean Theorem} \\ AB^2 &= 8^2 + 15^2 \\ &= 289 \\ AB &= \sqrt{289} = 17 \\ \text{Perimeter of } \triangle ABC &= AB + BC + AC \\ &= 17 + 15 + 8 = 40 \end{aligned}$$

2. A



Let  $BD = h$  and let  $AC = b$ .  
If  $BD$  was increased by 50 percent, the new  $BD$  will be  $h + 0.5h$ , or  $1.5h$ .  
If  $AC$  was reduced by 50 percent, the new  $AC$  will be  $b - 0.5b$ , or  $0.5b$ .

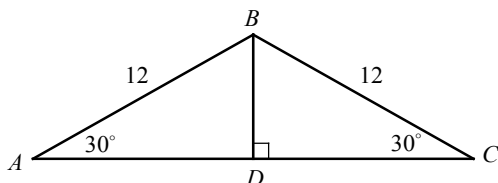
$$\text{The new area of } \triangle ABC = \frac{1}{2} (\text{new } AC) \times (\text{new } BD)$$

$$= \frac{1}{2}(0.5b)(1.5h) = \frac{1}{2}(0.75bh)$$

Because the area of the triangle before change was

$$\frac{1}{2}(bh), \text{ the area has decreased by 25 percent.}$$

3. C



$\triangle ABD$  and  $\triangle CBD$  are  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles.

In a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, the hypotenuse is twice as long as the shorter leg and the longer leg is  $\sqrt{3}$  times as long as the shorter leg.

$$AB = 2BD$$

$$12 = 2BD$$

$$6 = BD$$

$$AD = \sqrt{3}BD$$

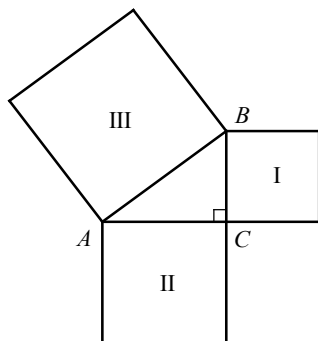
$$AD = \sqrt{3}(6) = 6\sqrt{3}$$

$$AC = 2AD = 12\sqrt{3}$$

$$\text{Area of } \triangle ABC = \frac{1}{2}AC \cdot BD = \frac{1}{2}(12\sqrt{3})(6)$$

$$= 36\sqrt{3}$$

4. C



The area of a square is the square of the length of any side.

The area of square region I =  $BC^2 = 80$ .

The area of square region II =  $AC^2 = 150$ .

The area of square region III =  $AB^2$

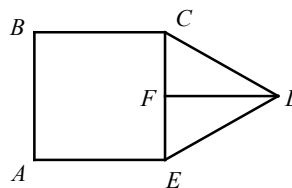
$$AB^2 = BC^2 + AC^2 \quad \text{Pythagorean Theorem}$$

$$= 80 + 150 = 230$$

Therefore, the area of square region III is 230.

### Chapter 17 Practice Test

1. B



If the area of square  $ABCD$  is  $4x^2$ , the length of the side of square  $ABCD$  is  $2x$ .

Drawing  $\overline{DF}$ , a perpendicular bisector of  $\overline{CE}$ , makes two  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles,  $\triangle CDF$  and  $\triangle EDF$ .

$$CE = 2x$$

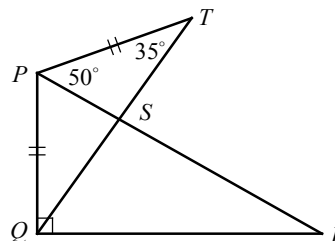
$$CF = \frac{1}{2}CE = \frac{1}{2}(2x) = x$$

$$DF = \sqrt{3}CF = \sqrt{3}x$$

$$\text{Area of } \triangle CDE = \frac{1}{2}CE \cdot DF = \frac{1}{2}(2x)(\sqrt{3}x)$$

$$= \sqrt{3}x^2$$

2. A



$$\overline{PQ} \cong \overline{PT} \quad \text{Given}$$

$$m\angle PQT = m\angle T = 35 \quad \text{Isosceles } \triangle \text{ Theorem}$$

$$m\angle PQT + m\angle T + m\angle QPT \quad \text{Angle Sum Theorem}$$

$$= 180$$

$$35 + 35 + m\angle QPT = 180 \quad \text{Substitution}$$

$$m\angle QPT = 110$$

$$m\angle QPT \quad \text{Angle Addition Postulate}$$

$$= m\angle QPR + m\angle RPT$$

$$110 = m\angle QPR + 50 \quad \text{Substitution}$$

$$60 = m\angle QPR$$

$$\overline{PQ} \perp \overline{QR} \quad \text{Given}$$

$$m\angle PQR = 90 \quad \text{Definition of Right } \angle$$

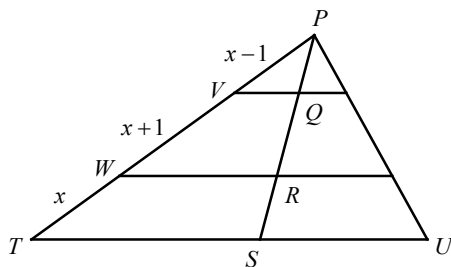
$$m\angle PQR + m\angle QPR + m\angle R \quad \text{Angle Sum Theorem}$$

$$= 180$$

$$90 + 60 + m\angle R = 180 \quad \text{Substitution}$$

$$m\angle R = 30$$

3. B



Note: Figure not drawn to scale.

Since  $\overline{VQ} \parallel \overline{WR} \parallel \overline{TS}$ ,  $\frac{PT}{PS} = \frac{x}{RS}$ .

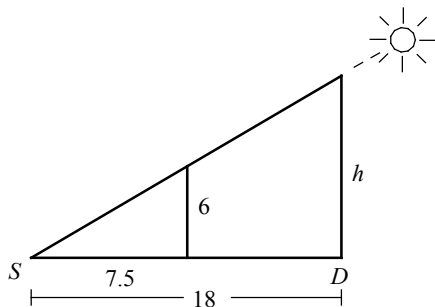
$$\frac{(x-1) + (x+1) + x}{15} = \frac{x}{RS} \quad \text{Substitution}$$

$$\frac{3x}{15} = \frac{x}{RS} \quad \text{Simplify.}$$

$$3x(RS) = 15x \quad \text{Cross Products}$$

$$RS = 5$$

4. C



Note: Figure not drawn to scale.

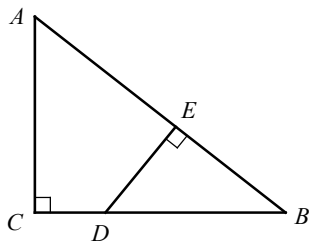
Let  $h$  = the length of the pole.

$$\frac{6}{7.5} = \frac{h}{18}$$

$$7.5h = 6 \times 18 \quad \text{Cross Products}$$

$$h = 14.4$$

5. D



- $m\angle C = m\angle BED$  All right  $\angle$ s are equal.
- $m\angle B = m\angle B$  Reflexive Property
- $\triangle ABC \sim \triangle DBE$  AA Similarity Postulate

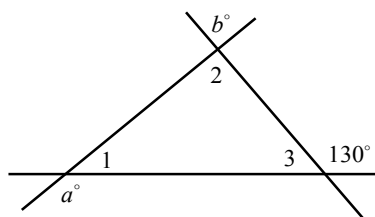
$$\frac{AC}{BC} = \frac{DE}{BE} \quad \text{AA Similarity Postulate}$$

$$\frac{12}{15} = \frac{8}{BE} \quad \text{Substitution}$$

$$12BE = 15 \times 8 \quad \text{Cross Products}$$

$$BE = 10$$

6. A



$$m\angle 1 + m\angle 2 + m\angle 3 = 180 \quad \text{Angle Sum Theorem}$$

$$a + m\angle 1 = 180 \quad \text{Straight } \angle \text{ measures } 180.$$

$$m\angle 1 = 180 - a$$

$$m\angle 2 = b \quad \text{Vertical } \angle \text{s are } \cong.$$

$$130 + m\angle 3 = 180 \quad \text{Straight } \angle \text{ measures } 180.$$

$$m\angle 3 = 50$$

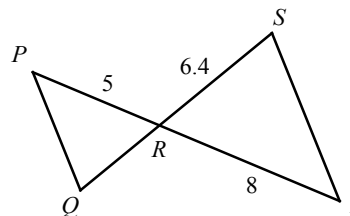
$$180 - a + b + 50 = 180 \quad \text{Substitution}$$

$$230 - a + b = 180$$

$$-a + b = -50$$

$$a - b = 50$$

7. 10.4



$$\overline{PQ} \parallel \overline{ST} \quad \text{Given}$$

$$m\angle P = m\angle T \quad \text{If } \overline{PQ} \parallel \overline{ST}, \text{ alternate interior } \angle \text{s are } \cong.$$

$$m\angle PRQ = m\angle TRS \quad \text{Vertical } \angle \text{s are } \cong.$$

$$\triangle PRQ \sim \triangle TRS \quad \text{AA Similarity Postulate}$$

$$\frac{PR}{TR} = \frac{RQ}{RS} \quad \text{AA Similarity Postulate}$$

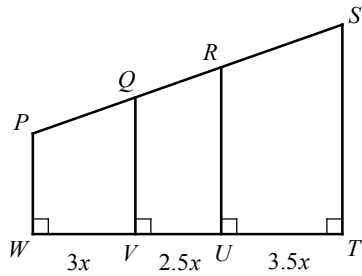
$$\frac{5}{8} = \frac{RQ}{6.4} \quad \text{Substitution}$$

$$8RQ = 5 \times 6.4 \quad \text{Cross Products}$$

$$RQ = 4$$

$$QS = SR + RQ = 6.4 + 4 = 10.4$$

8. 45



In the figure above,  $\overline{PW} \parallel \overline{QV} \parallel \overline{RU} \parallel \overline{ST}$ ,  
because they are all perpendicular to  $\overline{TW}$ .

Therefore,  $\frac{PS}{WT} = \frac{QR}{VU}$ .

$$\frac{162}{3x + 2.5x + 3.5x} = \frac{QR}{2.5x} \quad \text{Substitution}$$

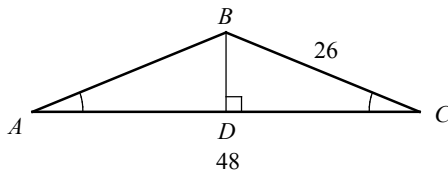
$$\frac{162}{9x} = \frac{QR}{2.5x} \quad \text{Simplify.}$$

$$9x(QR) = 162(2.5x) \quad \text{Cross Products}$$

$$9x(QR) = 405x \quad \text{Simplify.}$$

$$QR = 45$$

9. 240



Draw  $\overline{BD}$  perpendicular to  $\overline{AC}$ . Since  $\triangle ABC$  is  
an isosceles triangle,  $\overline{BD}$  bisects  $\overline{AC}$ .

Therefore,  $AD = CD = \frac{1}{2}AC = \frac{1}{2}(48) = 24$ .

$$CD^2 + BD^2 = BC^2 \quad \text{Pythagorean Theorem}$$

$$24^2 + BD^2 = 26^2$$

$$576 + BD^2 = 676$$

$$BD^2 = 100$$

$$BD = 10$$

$$\text{Area of } \triangle ABC = \frac{1}{2}(AC)(BD).$$

$$= \frac{1}{2}(48)(10)$$

$$= 240$$