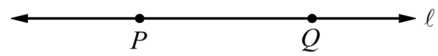
CHAPTER 16
Lines and Angles

16-1. Lines, Segments, and Rays

A **line** is a straight arrangement of points and extends in two directions without ending.

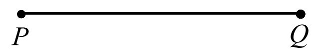
A line is often named by a lower-case script letter. If the names of two points on a line are known, then the line can be named by those points.

Written as: line ℓ, line PQ, or $\overleftrightarrow{PQ}$.



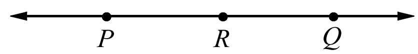
A **segment** is a part of a line and consists of two endpoints and all points in between.

Written as: segment PQ, or $\overline{PQ}$.



A **ray** is a part of a line. It has one endpoint and extends forever in one direction.

Written as: ray PQ or $\overrightarrow{PQ}$.



Two rays $\overrightarrow{RP}$ and $\overrightarrow{RQ}$ are called opposite rays if points R, P, and Q are collinear and R is between P and Q.

The length of $\overline{PQ}$, written as PQ, is the distance between the point P and point Q.

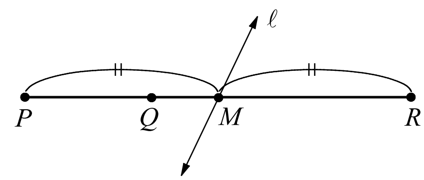
Segment Addition Postulate

If Q is between P and R, then $PQ + QR = PR$.

Definition of Midpoint

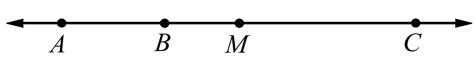
If M is the **midpoint** of $\overline{PR}$, then $PM = MR = \frac{1}{2}PR$.

A **segment bisector** is a line or a segment that intersects a segment at its midpoint.



Line ℓ is a segment bisector.

Example 1 □ Points A, B, M and C lie on the line as shown below. Point M is the midpoint of $\overline{AC}$.



a. Which ray is opposite to ray BC?

b. If $BM = 6$ and $AB = \frac{2}{3}MC$, what is the length of AM?

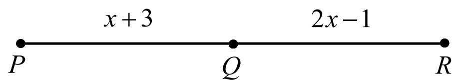
Solution □ a. Ray BA

b. Let $AB = x$.

$AM = AB + BM = x + 6$	Segment addition postulate
$AM = MC$	Definition of midpoint
$x + 6 = \frac{3}{2}x$	Substitution. If $AB = \frac{2}{3}MC$, $MC = \frac{3}{2}AB = \frac{3}{2}x$.
$x = 12$	Solve for x.
$AM = x + 6 = 12 + 6 = 18$	Substitute and simplify.

Exercises - Lines, Segments, and Rays

1



In the figure above, Q is the midpoint of PR. If $PQ = x+3$ and $QR = 2x-1$, what is the length of segment PR?

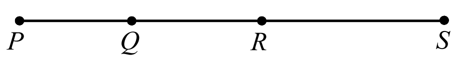
A) 4

B) 7

C) 11

D) 14

2



Note: Figure not drawn to scale.

On the segment PS above, $PR = 12$, $QS = 16$, and $QR = \frac{1}{3}PS$. What is the length of PS?

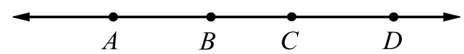
A) 19

B) 20

C) 21

D) 22

3



In the figure above, which of the following are opposite rays?

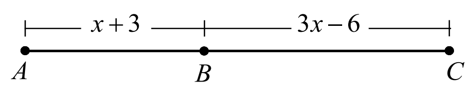
A) Ray AB and Ray CD

B) Ray CA and Ray CD

C) Ray DA and Ray AD

D) Ray CA and Ray BD

4



Note: Figure not drawn to scale.

In the figure above, $AB = \frac{2}{3}BC$. What is the length of AC?

A) 15

B) 18

C) 21

D) 25

16-2. Angles

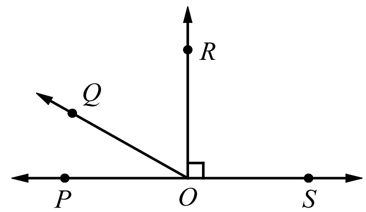
Angles are classified according to their measures.

An **acute angle** measures between 0 and 90. Ex. $\angle POQ$ and $\angle QOR$

A **right angle** measures 90. Ex. $\angle POR$ and $\angle SOR$

An **obtuse angle** measures between 90 and 180. Ex. $\angle QOS$

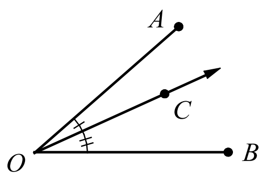
A **straight angle** measures 180. Ex. $\angle POS$



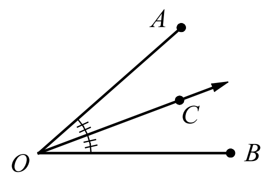
Angle Addition Postulate

If C is in the interior of $\angle AOB$, then $m\angle AOB = m\angle AOC + m\angle COB$.

An **angle bisector** divides an angle into two congruent angles.



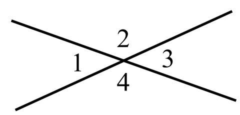
$m\angle AOB = m\angle AOC + m\angle COB$



If $\overrightarrow{OC}$ is the angle bisector of $\angle AOB$,
then $m\angle AOC = m\angle COB = \dfrac{1}{2} m\angle AOB$.

Special Pairs of Angles

When two lines intersect, they form two pairs of **vertical angles**.

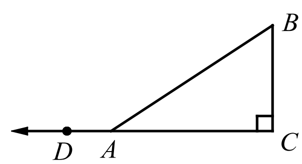


Vertical angles are congruent.

$\angle 1 \cong \angle 3$ $(m\angle 1 = m\angle 3)$ $\angle 2 \cong \angle 4$ $(m\angle 2 = m\angle 4)$

Two angles whose measures have a sum of 180 are called **supplementary angles**.

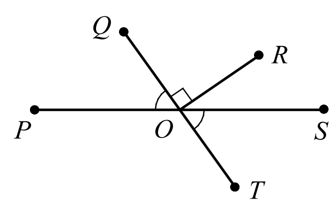
Two angles whose measures have a sum of 90 are called **complementary angles**.



$\angle DAB$ and $\angle BAC$ are supplementary.
$\angle B$ and $\angle BAC$ are complementary.

Example 1 □ In the figure shown at the right, $m\angle POQ = 55$.
Find the each of the following.

a. $m\angle SOT$ b. $m\angle ROT$ c. $m\angle POT$ d. $m\angle POR$

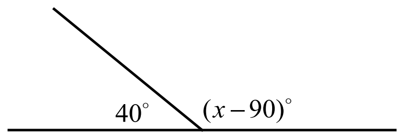


Solution □

a. $m\angle SOT = m\angle POQ = 55$	Vertical angles are congruent.
b. $m\angle QOR + m\angle ROT = 180$	Straight angle measures 180.
$90 + m\angle ROT = 180$	$m\angle QOR = 90$
$m\angle ROT = 90$	Solve for $m\angle ROT$.
c. $m\angle POQ + m\angle POT = 180$	Straight angle measures 180.
$55 + m\angle POT = 180$	$m\angle POQ = 55$
$m\angle POT = 125$	Solve for $m\angle POT$.
d. $m\angle POR = m\angle POQ + m\angle QOR$	Angle Addition Postulate
$m\angle POR = 55 + 90 = 145$	Substitution

Exercises - Angles

1



In the figure above, what is the value of x ?

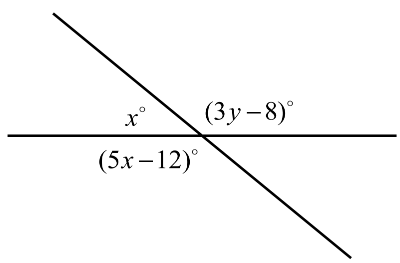
A) 140

B) 160

C) 190

D) 230

2



Note: Figure not drawn to scale.

In the figure above, what is the values of y ?

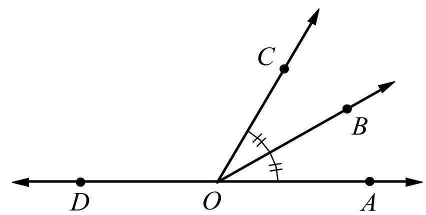
A) 52

B) 60

C) 68

D) 76

3



Note: Figure not drawn to scale.

In the figure above, ray OB bisects $\angle COA$. If $m\angle DOB = 11x+6$ and $m\angle COA = 8x-12$, what is the measure of $\angle DOC$?

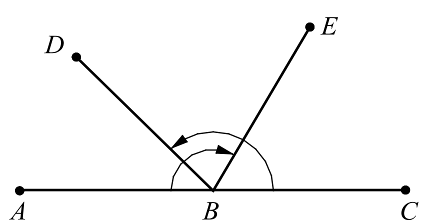
A) 92

B) 96

C) 102

D) 108

4



Note: Figure not drawn to scale.

In the figure above, $m\angle ABE = 120^\circ$ and $m\angle CBD = 135^\circ$. What is the measure of $\angle DBE$?

A) 63

B) 68

C) 75

D) 79

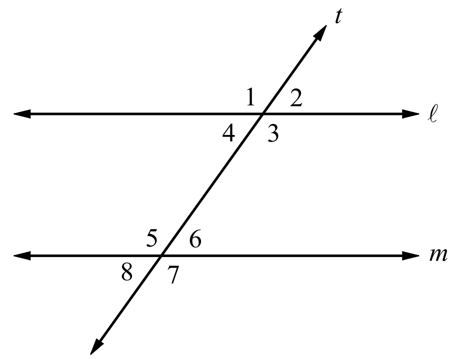
16-3. Parallel and Perpendicular Lines

For two parallel lines ℓ and m which are cut by the transversal t :

1) **Corresponding Angles** are equal in measure.

$m\angle 1 = m\angle 5 \quad m\angle 2 = m\angle 6$

$m\angle 3 = m\angle 7 \quad m\angle 4 = m\angle 8$



2) **Alternate Interior Angles** are equal in measure.

$m\angle 3 = m\angle 5 \quad m\angle 4 = m\angle 6$

3) **Alternate Exterior Angles** are equal in measure.

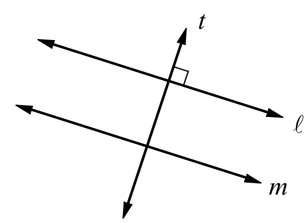
$m\angle 1 = m\angle 7 \quad m\angle 2 = m\angle 8$

4) **Consecutive(Same Side) Interior Angles** are supplementary.

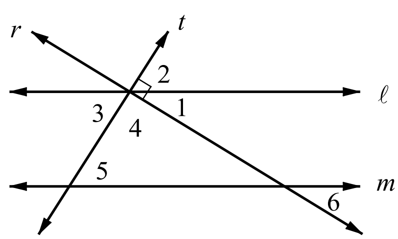
$m\angle 3 + m\angle 6 = 180^\circ \quad m\angle 4 + m\angle 5 = 180^\circ$

Theorem
In a plane, if a line is perpendicular to one of two parallel lines, it is also perpendicular to the other.

If $t \perp \ell$ and $\ell \parallel m$, then $t \perp m$.



Example 1 □ In the figure below, $\ell \parallel m$, $r \perp t$ and $m\angle 1 = 32$. Lines ℓ, r, and t intersect at one point. Find $m\angle 2$, $m\angle 3$, $m\angle 4$, and $m\angle 5$.

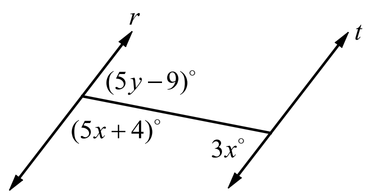


Solution □

$m\angle 1 + m\angle 2 = 90$	A right angle measures 90.
$32 + m\angle 2 = 90$	Substitution
$m\angle 2 = 58$	Solve for $m\angle 2$.
$m\angle 2 = m\angle 3 = 58$	Vertical angles are $\cong$.
$m\angle 1 + m\angle 4 + m\angle 3 = 180$	A straight angle measures 180.
$32 + m\angle 4 + 58 = 180$	Substitution
$m\angle 4 = 90$	Solve for $m\angle 4$.
$m\angle 3 = m\angle 5 = 58$	Alternate Interior $\angle s$ are $\cong$.
$m\angle 1 = m\angle 6 = 32$	Corresponding $\angle s$ are $\cong$.

Exercises - Parallel and Perpendicular Lines

1



Note: Figure not drawn to scale

In the figure above, $r \parallel t$. What is the value of $x+y$?

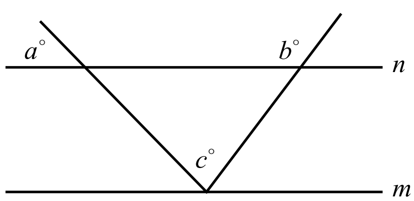
A) 37

B) 40

C) 43

D) 46

2



In the figure above, $m \parallel n$. If $a=50$ and $b=120$, what is the value of c ?

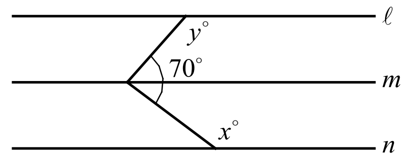
A) 50

B) 60

C) 70

D) 80

3



Note: Figure not drawn to scale.

In the figure above, lines ℓ , m , and n are parallel. What is the value of $x+y$?

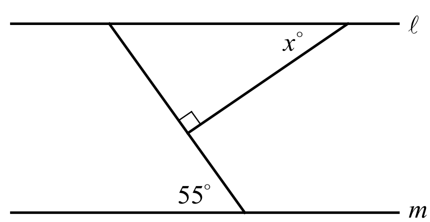
A) 160

B) 200

C) 230

D) 290

4



In the figure above, $\ell \parallel m$. What is the value of x ?

A) 30

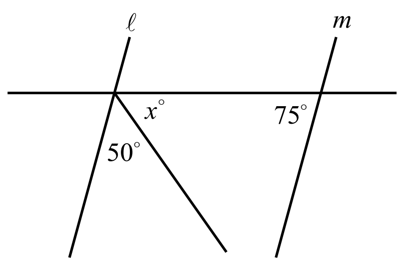
B) 35

C) 40

D) 45

Chapter 16 Practice Test

1



Note: Figure not drawn to scale.

In the figure above, $\ell \parallel m$. What is the value of x?

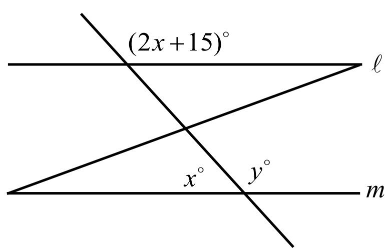
A) 45

B) 50

C) 55

D) 60

2



Note: Figure not drawn to scale.

In the figure above, $\ell \parallel m$. What is the value of y?

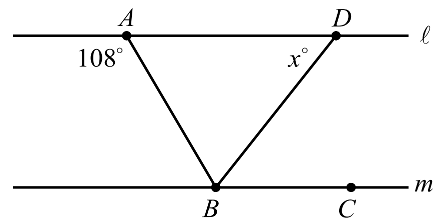
A) 120

B) 125

C) 130

D) 135

3



Note: Figure not drawn to scale.

In the figure above, lines ℓ and m are parallel and $\overline{BD}$ bisects $\angle ABC$. What is the value of x?

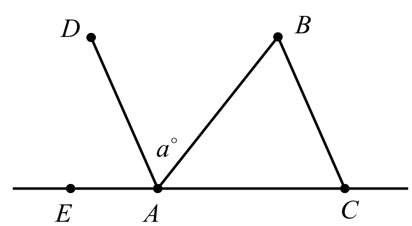
A) 54

B) 60

C) 68

D) 72

4



In the figure above, $\overline{DA} \parallel \overline{BC}$ and $\overline{AB}$ bisects $\angle DAC$. What is the measure of $\angle BCA$ in terms of a?

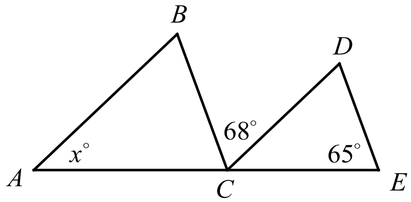
A) $180 - a$

B) $2a - 180$

C) $180 - 2a$

D) $2a - 90$

5



Note: Figure not drawn to scale.

In the figure above, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{DE}$. What is the value of x?

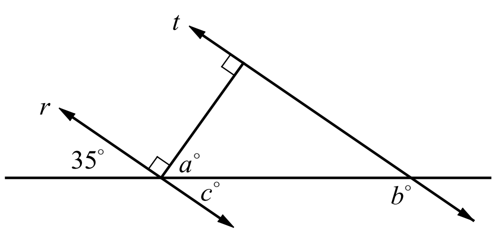
A) 47

B) 51

C) 55

D) 57

6



In the figure above, $r \parallel t$. What is the value of $a+b$?

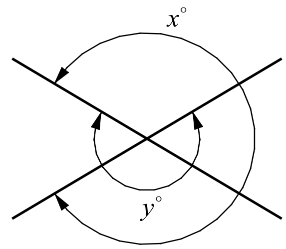
A) 160

B) 175

C) 185

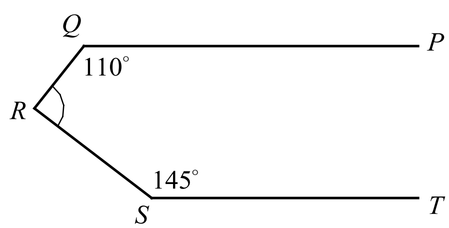
D) 200

7



In the figure above, what is the value of $x+y$?

8



Note: Figure not drawn to scale.

In the figure above, $\overline{PQ}$ is parallel to $\overline{ST}$. What is the measure of $\angle QRS$?

Answer Key

Section 16-1

1. D 2. C 3. B 4. D

Section 16-2

1. D 2. A 3. B 4. C

Section 16-3

1. A 2. C 3. D 4. B

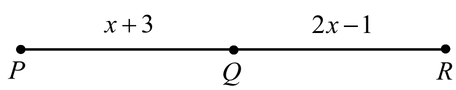
Chapter 16 Practice Test

1. C 2. B 3. A 4. C 5. A
6. D 7. 540 8. 105

Answers and Explanations

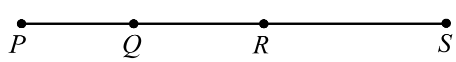
Section 16-1

1. D



$PQ = QR$	Definition of Midpoint
$x+3 = 2x-1$	Substitution
$x+3-x = 2x-1-x$	Subtract x from each side.
$3 = x-1$	Simplify.
$4 = x$	
$PR = PQ+QR$	Segment Addition Postulate
$\quad = x+3+2x-1$	Substitution
$\quad = 3x+2$	
$\quad = 3(4)+2 = 14$	$x = 4$

2. C



Note: Figure not drawn to scale.

Let $PS = x$, then $QR = \frac{1}{3}PS = \frac{1}{3}x$.

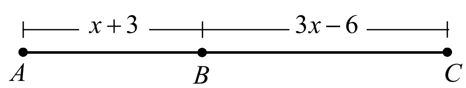
$PR = PQ+QR$	Segment Addition Postulate
$12 = PQ + \frac{1}{3}x$	$PR = 12$ and $QR = \frac{1}{3}x$
$PQ = 12 - \frac{1}{3}x$	Solve for PQ.
$QS = QR+RS$	Segment Addition Postulate
$16 = \frac{1}{3}x + RS$	$QS = 16$ and $QR = \frac{1}{3}x$
$RS = 16 - \frac{1}{3}x$	Solve for RS.
$PS = PQ+QR+RS$	Segment Addition Postulate
$x = (12-\frac{1}{3}x) + \frac{1}{3}x + (16-\frac{1}{3}x)$	Substitution
$x = 28 - \frac{1}{3}x$	Simplify.
$\frac{4}{3}x = 28$	Add $\frac{1}{3}x$ to each side .
$\frac{3}{4}\cdot\frac{4}{3}x = \frac{3}{4}\cdot 28$	Multiply $\frac{3}{4}$ by each side.
$x = 21$	

Therefore, $PS = x = 21$.

3. B

Ray CA and Ray CD are opposite rays, because points A, C, and D are collinear and C is between A and D.

4. D



Note: Figure not drwan to scale.

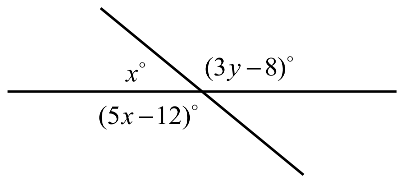
$AB = \frac{2}{3}BC$	Given
$x+3 = \frac{2}{3}(3x-6)$	Substitution
$x+3 = 2x-4$	Simplify.
$7 = x$	Solve for x .
$AC = AB+BC$	Segment Addition Postulate
$\quad = x+3+3x-6$	Substitution
$\quad = 4x-3$	Simplify.
$\quad = 4(7)-3$	$x = 7$
$\quad = 25$	

Section 16-2

1. D

$40 + x - 90 = 180$	Straight $\angle$ measures 180.
$x - 50 = 180$	Simplify.
$x - 50 + 50 = 180 + 50$	Add 50 to each side.
$x = 230$	

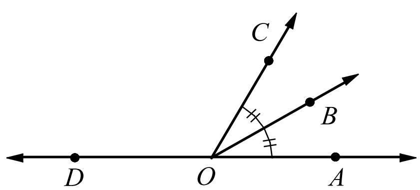
2. A



Note: Figure not drawn to scale.

$x+5x-12=180$	Straight $\angle$ measures 180.
$6x-12=180$	
$6x=192$	
$x=32$	
$x+3y-8=180$	Straight $\angle$ measures 180.
$32+3y-8=180$	$x=32$
$24+3y=180$	Simplify.
$24+3y-24=180-24$	
$3y=156$	
$y=52$	

3. B



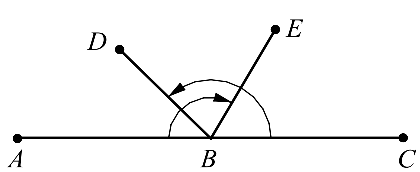
Note: Figure not drawn to scale.

$m\angle BOA=\dfrac{1}{2}m\angle COA$	Definition of $\angle$ bisector
$m\angle BOA=\dfrac{1}{2}(8x-12)$	Substitution
$m\angle BOA=4x-6$	Simplify.
$m\angle DOB+m\angle BOA=180$	Straight $\angle$ measures 180.
$11x+6+4x-6=180$	Substitution
$15x=180$	Simplify.
$x=12$	

Thus, $m\angle COA=8x-12=8(12)-12=84$.

$m\angle DOC+m\angle COA=180$	Straight $\angle$ measures 180.
$m\angle DOC+84=180$	$m\angle COA=84$
$m\angle DOC=96$	

4. C



Note: Figure not drawn to scale.

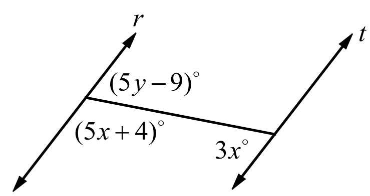
Let $m\angle DBE=x$

$m\angle ABE$	
$=m\angle ABD+m\angle DBE$	Angle Addition Postulate
$120=m\angle ABD+x$	Substitution
$120-x=m\angle ABD$	
$m\angle ABD+m\angle CBD=180$	Straight $\angle$ measures 180.
$120-x+135=180$	Substitution
$255-x=180$	Simplify.
$x=75$	

Therefore, $m\angle DBE=x=75$.

Section 16-3

1. A

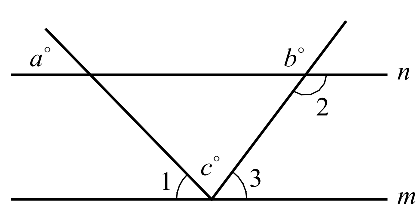


Note: Figure not drawn to scale

$5x+4+3x=180$	If $r \parallel t$, consecutive interior $\angle s$ are supplementary.
$8x+4=180$	Simplify.
$8x=176$	
$x=22$	
$5x+4+5y-9=180$	Straight $\angle$ measures 180.
$5x-5+5y=180$	Simplify.
$5(22)-5+5y=180$	$x=22$
$110-5+5y=180$	Simplify.
$105+5y=180$	Simplify.
$5y=75$	Simplify.
$y=15$	

Therefore, $x+y=22+15=37$.

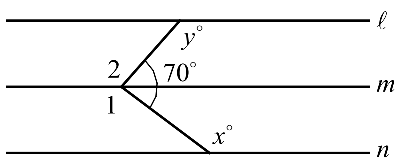
2. C



$m\angle 1=a$	If $m \parallel n$, corresponding $\angle s$ are $\cong$.
$m\angle 1=50$	$a=50$
$m\angle 2=b$	Vertical $\angle s$ are $\cong$.
$m\angle 2=120$	$b=120$

$m\angle 2 + m\angle 3 = 180$	If $m \parallel n$, consecutive interior $\angle s$ are supplementary.
$120 + m\angle 3 = 180$	$m\angle 2 = 120$
$m\angle 3 = 60$	
$m\angle 1 + c + m\angle 3 = 180$	Straight $\angle$ measures 180.
$50 + c + 60 = 180$	$m\angle 1 = 50$ and $m\angle 3 = 60$
$c + 110 = 180$	Simplify.
$c = 70$	

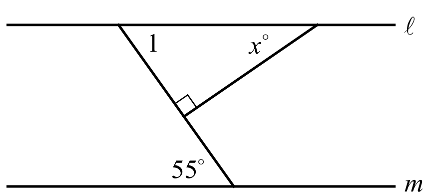
3. D



Note: Figure not drawn to scale.

$m\angle 1 = x$	If $m \parallel n$, alternate interior $\angle s$ are $\cong$.
$m\angle 2 = y$	If $\ell \parallel m$, alternate interior $\angle s$ are $\cong$.
$m\angle 1 + m\angle 2 + 70 = 360$	There are 360° in a circle.
$x + y + 70 = 360$	$m\angle 1 = x$ and $m\angle 2 = y$
$x + y = 290$	

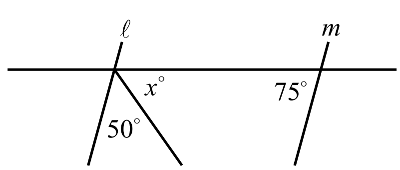
4. B



$m\angle 1 = 55$	If $\ell \parallel m$, alternate interior $\angle s$ are $\cong$.
$m\angle 1 + x = 90$	The acute $\angle s$ of a right triangle are complementary.
$55 + x = 90$	$m\angle 1 = 55$
$x = 35$	

Chapter 16 Practice Test

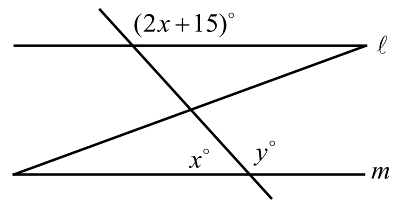
1. C



Note: Figure not drawn to scale.

$50 + x + 75 = 180$	If $\ell \parallel m$, consecutive interior $\angle s$ are supplementary.
$125 + x = 180$	Simplify.
$x = 55$	

2. B

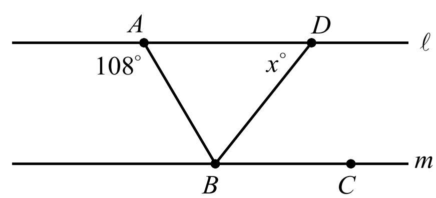


Note: Figure not drwan to scale.

$y = 2x + 15$	If $\ell \parallel m$, consecutive interior $\angle s$ are supplementary.
$x + y = 180$	Straight $\angle$ measures 180.
$x + (2x + 15) = 180$	$y = 2x + 15$
$3x + 15 = 180$	Simplify.
$3x = 165$	
$x = 55$	

Therefore, $y = 2x + 15 = 2(55) + 15 = 125$.

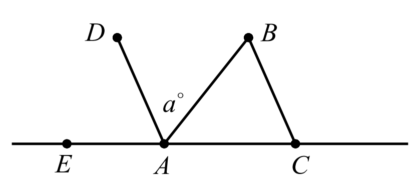
3. A



Note: Figure not drawn to scale.

$m\angle ABC = 108$	If $\ell \parallel m$, alternate interior $\angle s$ are $\cong$.
$m\angle DBC = \frac{1}{2} m\angle ABC$	Definition of $\angle$ bisector
$m\angle DBC = \frac{1}{2}(108)$	$m\angle ABC = 108$
$m\angle DBC = 54$	Simplify.
$x = m\angle DBC$	If $\ell \parallel m$, alternate interior $\angle s$ are $\cong$.
$x = 54$	$m\angle DBC = 54$

4. C

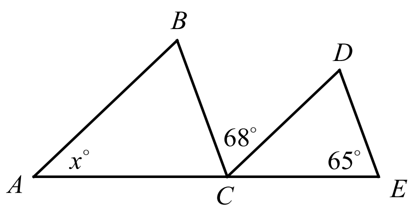


$m\angle BAC = m\angle DAB$	Definition of $\angle$ bisector
$m\angle BAC = a$	$m\angle DAB = a$

Since straight angles measure 180,
$m\angle DAE + m\angle DAB + m\angle BAC = 180$.

$m\angle DAE + a + a = 180$	$m\angle DAB = m\angle BAC = a$
$m\angle DAE = 180 - 2a$	Subtract $2a$.
$m\angle BCA = m\angle DAE$	If $DA \parallel BC$, corresponding $\angle s$ are $\cong$.
$m\angle BCA = 180 - 2a$	$m\angle DAE = 180 - 2a$

5. A



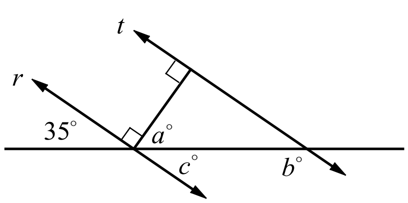
Note: Figure not drawn to scale.

$m\angle BCA = m\angle DEC$	If $DE \parallel BC$, corresponding $\angle s$ are $\cong$.
$m\angle BCA = 65$	$m\angle DEC = 65$
$m\angle DCE = x$	If $AB \parallel CD$, corresponding $\angle s$ are $\cong$.

Since straight angles measure 180,
$m\angle BCA + m\angle BCD + m\angle DCE = 180$.

$65 + 68 + x = 180$	Substitution
$133 + x = 180$	Simplify.
$x = 47$	

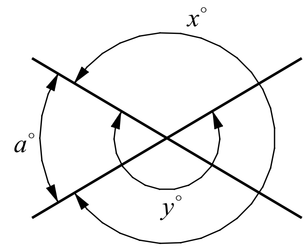
6. D



$c = 35$	Vertical $\angle s$ are $\cong$.
$a + c = 90$	$\angle a$ and $\angle c$ are complementary.
$a + 35 = 90$	$c = 35$
$a = 55$	
$b + c = 180$	If $r \parallel t$, consecutive interior $\angle s$ are supplementary.
$b + 35 = 180$	$c = 35$
$b = 145$	

Therefore, $a + b = 55 + 145 = 200$.

7. 540

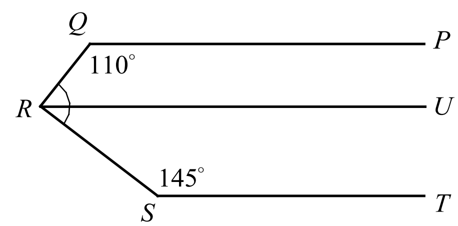


Draw $\angle a$.

$x + a = 360$	360° in a circle.
$x = 360 - a$	Subtract a from each side.
$y - a = 180$	Straight $\angle$ measures 180.
$y = 180 + a$	Add a to each side.

Therefore, $x + y = (360 - a) + (180 + a) = 540$.

8. 105



Note: Figure not drawn to scale.

Draw $\overline{RU}$, which is parallel to $\overline{PQ}$ and $\overline{ST}$.

If two lines are parallel, then the consecutive interior angles are supplementary. Therefore, $m\angle PQR + m\angle QRU = 180$ and $m\angle RST + m\angle URS = 180$.

$110 + m\angle QRU = 180$	$m\angle PQR = 110$
$m\angle QRU = 70$	Subtract 110.
$145 + m\angle URS = 180$	$m\angle RST = 145$
$m\angle URS = 35$	Subtract 145.

By the Angle Addition Postulate,
$m\angle QRS = m\angle QRU + m\angle URS$.
Substituting 70 for $m\angle QRU$ and 35 for $m\angle QRU$ gives $m\angle QRS = 70 + 35 = 105$.