Answer Key

Section 15-1

1. B	2. C	3. B	4. D	5. C

Section 15-2

1. B	2. C	3. D	4. A

Section 15-3

1. A	2. C	3. B	4. D

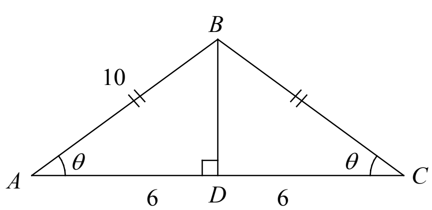
Chapter 15 Practice Test

1. D	2. C	3.C	4. B	5. D
6. D	7. $\frac{5}{13}$	8. 9	9. 10.5	

Answers and Explanations

Section 15-1

1. B



Draw a perpendicular segment from B to the opposite side AC. Let the perpendicular segment intersect side AC at D. Because the triangle is isosceles, a perpendicular segment from the vertex to the opposite side bisects the base and creates two congruent right triangles.

Therefore, $AD = \frac{1}{2}AC = \frac{1}{2}(12) = 6$.

In right ΔABD,

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{6}{10} = 0.6$.

2. C

$AB^2 = BD^2 + AD^2$ Pythagorean Theorem
$10^2 = BD^2 + 6^2$
$100 = BD^2 + 36$
$64 = BD^2$
$8 = BD$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{8}{10} = 0.8$

3. B

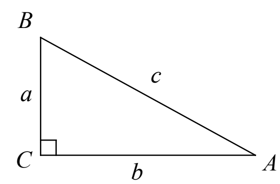
$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BD}{AD} = \frac{8}{6} = \frac{4}{3}$

4 . D

If x and y are acute angles and $\cos x^\circ = \sin y^\circ$, $x + y = 90$ by the complementary angle theorem.

$(3a-14)+(50-a)=90$ $x = 3a-14$, $y = 50-a$
$2a+36=90$ Simplify.
$2a=54$
$a=27$

5. C



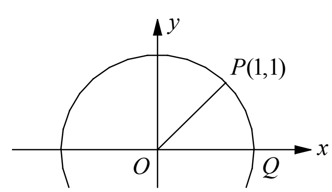
I. $\sin A = \frac{\text{opposite of } \angle A}{\text{hypotenuse}} = \frac{a}{c}$
Roman numeral I is true.

II. $\cos B = \frac{\text{adjacent of } \angle B}{\text{hypotenuse}} = \frac{a}{c}$
Roman numeral II is true.

III. $\tan A = \frac{\text{opposite of } \angle A}{\text{adjacent of } \angle A} = \frac{a}{b}$
Roman numeral III is false.

Section 15-2

1. B



The graph shows $P(x, y) = P(1,1)$. Thus, $x = 1$ and $y = 1$. Use the distance formula to find the length of radius OA.

$OA = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$ or $\sin\theta = \frac{\sqrt{2}}{2}$

Therefore, the measure of $\angle POQ$ is 45°,

which is equal to $45(\frac{\pi}{180}) = \frac{\pi}{4}$ radians.

Thus, $k = \frac{1}{4}$.

2. C

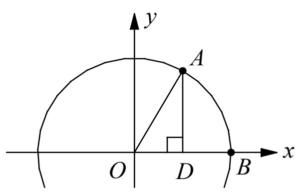
Use the complementary angle theorem.

$\cos(\theta) = \sin(90^\circ - \theta)$, or $\cos(\theta) = \sin(\frac{\pi}{2} - \theta)$

Therefore, $\cos(\frac{\pi}{8}) = \sin(\frac{\pi}{2} - \frac{\pi}{8}) = \sin(\frac{3\pi}{8})$.

All the other answer choices have values different from $\cos(\frac{\pi}{8})$.

3. D

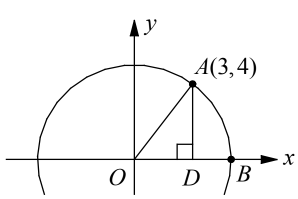


In ΔOAD, $\sin\frac{\pi}{3} = \sin 60^\circ = \frac{AD}{OA} = \frac{AD}{6}$.

Since $\sin 60^\circ = \frac{\sqrt{3}}{2}$, you get $\frac{AD}{6} = \frac{\sqrt{3}}{2}$.

Therefore, $2AD = 6\sqrt{3}$ and $AD = 3\sqrt{3}$.

4. A



Use the distance formula to find the length of OA.

$OA = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$\cos\angle AOD = \frac{OD}{OA} = \frac{3}{5}$

Section 15-3

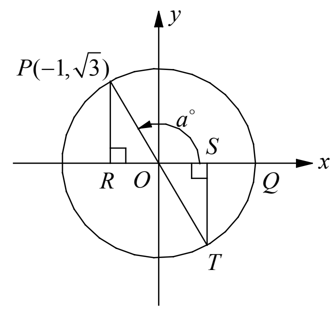
1. A

Draw segment PR, which is perpendicular to the x-axis. In right triangle POR, $x = -1$ and $y = \sqrt{3}$. To find the length of OP, use the Pythagorean theorem.

$OP^2 = PR^2 + OR^2 = (\sqrt{3})^2 + (-1)^2 = 4$

Which gives $OP = 2$.

$\cos a^\circ = \frac{x}{OP} = \frac{-1}{2}$

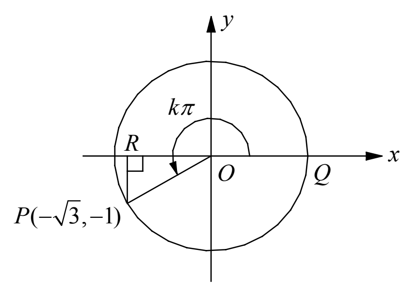


2. C

Since the terminal side of $(a+180)^\circ$ is OT, the value of $\cos(a+180)^\circ$ is equal to $\frac{OS}{OT}$.

$\frac{OS}{OT} = \frac{1}{2}$

3. B



Draw segment PR, which is perpendicular to the x-axis. In right triangle POR, $x = -\sqrt{3}$ and $y = -1$. To find the length of OP, use the Pythagorean theorem.

$OP^2 = PR^2 + OR^2 = (-1)^2 + (\sqrt{3})^2 = 4$

Which gives $OP = 2$.

Since $\sin\angle POR = \frac{y}{OP} = \frac{-1}{2}$, the measure of $\angle POR$ is equal to 30°, or $\frac{\pi}{6}$ radian.

$k\pi = \pi + \frac{\pi}{6} = \frac{7}{6}\pi$

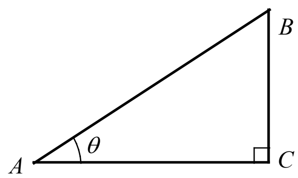
Therefore, $k = \frac{7}{6}$

4. D

$\tan(k\pi) = \tan(\frac{7}{6}\pi) = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

Chapter 15 practice Test

1. D



Note: Figure not drawn to scale.

In ΔABC, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC}$.

If $\tan\theta = \frac{3}{4}$, then $BC = 3$ and $AC = 4$.

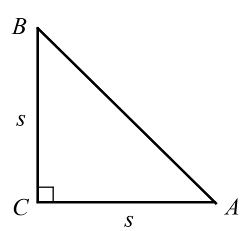
By the Pythagorean theorem,

$AB^2 = AC^2 + BC^2 = 4^2 + 3^2 = 25$, thus

$AB = \sqrt{25} = 5$.

$\sin\theta = \frac{BC}{AB} = \frac{3}{5}$

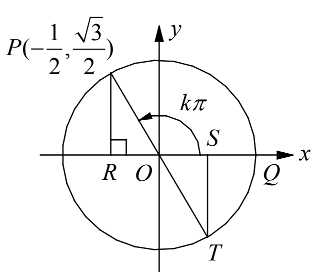
2. C



$\tan\angle A = \frac{\text{opposite side of } \angle A}{\text{adjacent side of } \angle A} = \frac{s}{s} = 1$

$= \frac{s}{s} = 1$

3. C



Draw segment PR, which is perpendicular to the x-axis. In right triangle POR, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$. To find the length of OP, use the Pythagorean theorem.

$OP^2 = PR^2 + OR^2 = (\frac{\sqrt{3}}{2})^2 + (\frac{-1}{2})^2 = \frac{3}{4} + \frac{1}{4} = 1$

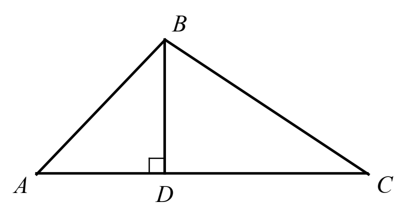
Which gives $OP = 1$. Thus, triangle OPR is 30°-60°-90° triangle and the measure of $\angle POR$ is 60°, which is $\frac{\pi}{3}$ radian. Therefore, the measure of $\angle POQ$ is $\pi - \frac{\pi}{3}$, or $\frac{2\pi}{3}$ radian. If $\angle POQ$ is $k\pi$ radians then k is equal to $\frac{2}{3}$.

4. B

Since the terminal side of $(k+1)\pi$ is OT, the value of $\cos(k+1)\pi$ is equal to $\frac{OS}{OT}$.

$\frac{OS}{OT} = \frac{1}{2}$

5. D



Area of triangle $ABC = \frac{1}{2}(AC)(BD)$

Check each answer choice.

A) $\frac{1}{2}(AB\cos\angle A + BC\cos\angle C)(AB\cos\angle ABD)$

$= \frac{1}{2}(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC})(AB \cdot \frac{BD}{AB})$

$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$

B) $\frac{1}{2}(AB\cos\angle A + BC\cos\angle C)(BC\sin\angle C)$

$= \frac{1}{2}(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC})(BC \cdot \frac{BD}{BC})$

$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$

C) $\frac{1}{2}(AB\sin\angle ABD + BC\sin\angle CBD)(AB\sin\angle A)$

$$= \frac{1}{2}(AB\cdot\frac{AD}{AB} + BC\cdot\frac{CD}{BC})(AB\cdot\frac{BD}{AB})$$
$$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$$

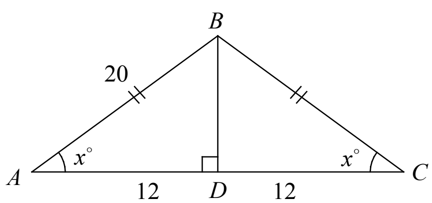
D) $\frac{1}{2}(AB\sin\angle ABD + BC\sin\angle CBD)(BC\cos\angle C)$

$$= \frac{1}{2}(AB\cdot\frac{AD}{AB} + BC\frac{CD}{BC})(BC\cdot\frac{CD}{BC})$$
$$= \frac{1}{2}(AD + CD)(CD) = \frac{1}{2}(AC)(CD)$$

Which does not represent the area of triangle ABC.

Choice D is correct.

6. D



Draw segment BD, which is perpendicular to side AC. Because the triangle is isosceles, a perpendicular segment from the vertex to the opposite side bisects the base and creates two congruent right triangles.

Therefore, $AD = \frac{1}{2}AC = \frac{1}{2}(24) = 12$.

By the Pythagorean theorem, $AB^2 = BD^2 + AD^2$

Thus, $20^2 = BD^2 + 12^2$.

$BD^2 = 20^2 - 12^2 = 256$

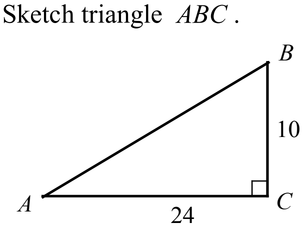
$BD = \sqrt{256} = 16$

In right ΔABD,

$\sin x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{16}{20} = \frac{4}{5}$.

7. $\frac{5}{13}$

Sketch triangle ABC.



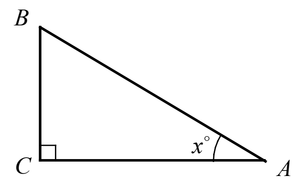
$AB^2 = BC^2 + AC^2$

$AB^2 = 10^2 + 24^2 = 676$

$AB = \sqrt{676} = 26$

$\sin A = \frac{10}{26} = \frac{5}{13}$

8. 9



$\cos x^\circ = \frac{AC}{AB} = \frac{3}{5}$

Let $AC = 3x$ and $AB = 5x$.

$AB^2 = BC^2 + AC^2$ Pythagorean Theorem

$(5x)^2 = 12^2 + (3x)^2$ $BC = 12$

$25x^2 = 144 + 9x^2$

$16x^2 = 144$

$x^2 = 9$

$x = \sqrt{9} = 3$

Therefore, $AC = 3x = 3(3) = 9$

9. 10.5

According to the complementary angle theorem, $\sin\theta = \cos(90 - \theta)$.

If $\sin(5x - 10)^\circ = \cos(3x + 16)^\circ$,

$3x + 16 = 90 - (5x - 10)$.

$3x + 16 = 90 - 5x + 10$

$3x + 16 = 100 - 5x$

$8x = 84$

$x = 10.5$