CHAPTER 14
Rational Expressions

14-1. Rational Expressions

A **rational expression** is an algebraic fraction whose numerator and denominator are polynomials. Any value of a variable that makes the denominator of a rational expression zero must be excluded from the domain of that variable.

Rule for Multiplying and Dividing Rational Expressions

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ and $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$, if the denominators are not zero.

Rule for Adding and Subtracting Rational Expressions

$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$ and $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$

The **least common denominator** (**LCD**) is the LCM of the denominators.

Use the following steps to add or subtract fractions with unlike denominators.

1. Find the LCD of the fractions.
2. Express each fraction as an equivalent fraction with the LCD as denominator.
3. Add or subtract the numerator, then simplify if necessary.

Example 1 □ Simplify.

a. $\frac{x^2-2x-8}{3x-6} \cdot \frac{4x-8}{x-4}$ b. $\frac{2x^2-8}{4x+12} \div \frac{x+2}{x+3}$ c. $\frac{2}{x^2} + \frac{3}{2x}$

Solution □ a. $\frac{x^2-2x-8}{3x-6} \cdot \frac{4x-8}{x-4}$

$= \frac{\cancel{(x-4)}(x+2)}{3\cancel{(x-2)}} \cdot \frac{4\cancel{(x-2)}}{\cancel{(x-4)}}$ Factor and cancel.

$= \frac{4(x+2)}{3}$ Simplify.

b. $\frac{2x^2-8}{4x+12} \div \frac{x+2}{x+3} = \frac{2x^2-8}{4x+12} \cdot \frac{x+3}{x+2}$ Multiply by $\frac{x+3}{x+2}$, the reciprocal of $\frac{x+2}{x+3}$.

$= \frac{2(x^2-4)}{4(x+3)} \cdot \frac{x+3}{x+2}$ Factor $2x^2-8$ and $4x+12$.

$= \frac{{}^{1}\cancel{2}\,\cancel{(x+2)}(x-2)}{{}^{2}\cancel{4}\,\cancel{(x+3)}} \cdot \frac{\cancel{x+3}}{\cancel{x+2}}$ Factor x^2-4 and cancel.

$= \frac{x-2}{2}$ Simplify.

c. $\frac{2}{x^2} + \frac{3}{2x} = \frac{2}{x^2} \cdot \frac{2}{2} + \frac{3}{2x} \cdot \frac{x}{x}$ The LCD is $2x^2$.

$= \frac{4}{2x^2} + \frac{3x}{2x^2}$ Simplify.

$= \frac{3x+4}{2x^2}$ Add the numerators.

The sum or difference of a polynomial and a fraction is called a **mixed expression**. An expression like $2-\frac{1}{x+9}$ is called a mixed expression because it contains the sum of monomial 2 and the rational expression $\frac{1}{x+9}$.

If a fraction has one or more fractions in the numerator or denominator, it is called a **complex fraction**. To simplify a complex fraction, express the fraction as a quotient using the $\div$ sign.

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}\text{, in which } bcd\neq 0.$$

Example 2 □ Simplify.

a. $\dfrac{3\frac{3}{4}\text{ ft}}{6\frac{2}{3}\text{ in}}$ b. $x-\dfrac{x+1}{x-3}$ c. $\dfrac{\frac{5x}{x-3}}{\frac{15}{x^2-9}}$

Solution □ a. $\dfrac{3\frac{3}{4}\text{ ft}}{6\frac{2}{3}\text{ in}}=\dfrac{3\frac{3}{4}\cancel{\text{ft}}}{6\frac{2}{3}\cancel{\text{in}}}\cdot\dfrac{12\cancel{\text{in}}}{1\cancel{\text{ft}}}$ Convert feet to inches. Divide by common units.

$$=\frac{\frac{15}{{}_1\cancel{4}}\cdot\frac{\cancel{12}^3}{1}}{\frac{20}{3}}=\frac{\frac{45}{1}}{\frac{20}{3}}$$ Express each term as an improper fraction.

$$=\frac{{}^9\cancel{45}}{1}\cdot\frac{3}{\cancel{20}_4}$$ $\dfrac{\frac{a}{b}}{\frac{c}{d}}=\dfrac{ad}{bc}$

$$=\frac{27}{4}\text{ or }6\frac{3}{4}$$ Simplify.

b. $x-\dfrac{x+1}{x-3}=\dfrac{x(x-3)}{x-3}-\dfrac{x+1}{x-3}$ The LCD is $x-3$. Multiply x by $\dfrac{x-3}{x-3}$.

$$=\frac{x(x-3)-(x+1)}{x-3}$$ Add the numerators.

$$=\frac{x^2-4x-1}{x-3}$$ Simplify the numerator.

c. $\dfrac{\frac{5x}{x-3}}{\frac{15}{x^2-9}}=\dfrac{5x}{x-3}\div\dfrac{15}{x^2-9}$ Rewrite as a division sentence.

$$=\frac{5x}{x-3}\times\frac{x^2-9}{15}$$ Multiply by the reciprocal, $\dfrac{x^2-9}{15}$.

$$=\frac{{}^1\cancel{5}x}{\cancel{x-3}}\times\frac{(x+3)\cancel{(x-3)}}{\cancel{15}^3}=\frac{x(x+3)}{3}$$ Factor and simplify.

Exercises - Rational Expressions

1

If $n \neq 4$, which of the following is equivalent to $\dfrac{n^2}{n-4}+\dfrac{4n}{4-n}$?

A) n

B) $\dfrac{n(n+4)}{n-4}$

C) $\dfrac{n}{n-4}$

D) $\dfrac{n+4}{n-4}$

2

If $a \neq \pm 1$, which of the following is equivalent to $\dfrac{a}{a^2-1}-\dfrac{1}{a+1}$?

A) $\dfrac{1}{a-1}$

B) $\dfrac{1}{a+1}$

C) $\dfrac{2a-1}{a^2-1}$

D) $\dfrac{1}{a^2-1}$

3

If $y \neq -1$ and $y \neq 0$, which of the following is equivalent to $\dfrac{y^2-1}{1+\dfrac{1}{y}}$?

A) $\dfrac{y-1}{y}$

B) $y(y-1)$

C) $\dfrac{y}{y+1}$

D) $y-1$

4

If $x \neq \pm 1$, which of the following is equivalent to $\dfrac{1-\dfrac{1}{x+1}}{1+\dfrac{1}{x^2-1}}$?

A) $\dfrac{x-1}{x}$

B) $\dfrac{x+1}{x}$

C) $\dfrac{x-1}{x^2}$

D) $\dfrac{x+1}{x^2}$

5

If $x > 3$, which of the following is equivalent to $\dfrac{x-3}{\dfrac{1}{x+2}-\dfrac{1}{2x-1}}$?

A) $\dfrac{x-3}{(x+2)(2x-1)}$

B) $\dfrac{(x+2)(2x-1)}{x-3}$

C) $(x+2)(2x-1)$

D) $2x-1$

6

If $\dfrac{x^2-xy}{2x} \div \dfrac{x-y}{3x^2} = ax^2$, what is the value of a?

14-2. Solving Rational Equations

Rational equations are equations that contain rational expressions.

To solve rational equations, multiply the LCD of all the fractions on both sides of the equation. This will eliminate all of the fractions. Then solve the resulting equation.
You can also use cross products to solve rational equations, if both sides of the equation are single fractions.

Example 1 □ Solve each equation.

a. $\frac{x}{2x+5}-\frac{x+1}{4x+10}=\frac{1}{8}$.   b. $\frac{x-1}{2x}=\frac{x}{x+6}$

Solution □ a. $\frac{x}{2x+5}-\frac{x+1}{2(2x+5)}=\frac{1}{8}$ Factor.

$8(2x+5)(\frac{x}{2x+5}-\frac{x+1}{2(2x+5)})$ The LCD is $8(2x+5)$. Multiply each side by $8(2x+5)$.

$=8(2x+5)\cdot\frac{1}{8}$

$8x-4(x+1)=2x+5$ Distributive Property

$4x-4=2x+5$ Simplify.

$2x=9$ Simplify.

$x=\frac{9}{2}$ Solve for x.

b. $\frac{x-1}{2x}=\frac{x}{x+6}$ Both sides of the equation are single fractions.

$(x-1)(x+6)=2x\cdot x$ Cross multiply.

$x^2+5x-6=2x^2$ FOIL

$x^2-5x+6=0$ Make one side 0.

$(x-2)(x-3)=0$ Factor.

$x-2=0$ or $x-3=0$ Zero Product Property

$x=2$ or $x=3$ Solve for x.

A rational equation is **undefined** when the denominator is equal to zero. Multiplying both sides of a rational equation by the LCD can yield solutions with a denominator of zero. Such solutions are called **extraneous solutions**, which must be excluded from solutions to the original equation.

Example 2 □ Solve $\frac{9x}{x-2}-\frac{5x+8}{x-2}=6$.

Solution □ $(x-2)(\frac{9x}{x-2}-\frac{5x+8}{x-2})=6(x-2)$ Multiply each side by $x-2$.

$9x-(5x+8)=6x-12$ Distributive Property

$4x-8=6x-12$ Simplify.

$4=2x$ Simplify.

$2=x$ Solve for x.

If we substitute 2 for x in the original equation, we get undefined expressions.
So, this equation has no solution.

Exercises - Solving Rational Equations

1

$$\frac{x}{x-1}=\frac{x-2}{x+1}$$

What is the solution set of the equation above?

A) -2

B) $-\frac{1}{2}$

C) $\frac{1}{2}$

D) 2

2

$$\frac{x}{x-3}-2=\frac{4}{x-2}$$

What is the solution set of the equation above?

A) $\{0\}$

B) $\{2\}$

C) $\{0,\ 2\}$

D) $\{0,\ 4\}$

3

$$\frac{1}{x}-\frac{2}{x-2}=\frac{-4}{x^2-2x}$$

What is the solution set of the equation above?

A) -2

B) 0

C) 2

D) There is no solution to the equation.

4

$$\frac{3}{x^2-3x}+\frac{1}{3-x}=2$$

What is the solution set of the equation above?

A) $\{-\frac{1}{2}\}$

B) $\{3\}$

C) $\{-\frac{1}{2},\ 3\}$

D) $\{-\frac{1}{2},\ -3\}$

5

If $f(x)=\dfrac{1}{(x-a)^2-4(x-a)+4}$ is undefined when $x=6$, what is the value of a?

6

$$g(x)=\frac{1}{(x+3)^2-24(x+3)+144}$$

For what value of x is function g above undefined?

14-3. Direct, Inverse, and Joint Variations

A **direct variation** is an equation of the form $y = kx$, in which $k \neq 0$.
It is expressed as, y *varies directly as* x.
The graph of a direct variation is a straight line with slope k, and passes through the origin.

An **inverse variation** is an equation of the form $xy = k$ or $y = \frac{k}{x}$, in which $x \neq 0$.

It is expressed as, y *varies inversely as* x.

A **joint variation** is an equation of the form $z = kxy$, in which $k \neq 0$.
It is expressed as, z *varies jointly as* x *and* y.

Example 1 □ a. If y varies directly as x, and $y = 4$ when $x = 6$, find y when $x = 18$.

b. If w varies inversely as x, and $w = \frac{1}{3}$ when $x = 15$, find w when $x = 25$.

c. If z varies jointly as x and y, and $z = 18$ when $x = 2$ and $y = 3$, find z when $x = \frac{2}{3}$ and $y = \frac{5}{8}$.

Solution □ a.

$y = kx$	Direct variation formula
$4 = k(6)$	Replace y with 4 and x with 2.
$k = \frac{2}{3}$	Solve for k.
$y = \frac{2}{3}x$	Direct variation formula with $k = \frac{2}{3}$
$y = \frac{2}{3}(18) = 12$	Replace x with 18 and solve for y.

b.

$w = \frac{k}{x}$	Inverse variation formula
$\frac{1}{3} = \frac{k}{15}$	Replace w with $\frac{1}{3}$ and x with 15.
$k = 5$	Solve for k.
$w = \frac{5}{x}$	Inverse variation formula with $k = 5$
$w = \frac{5}{25} = \frac{1}{5}$	Replace x with 25 and solve for w.

c.

$z = kxy$	Joint variation formula
$18 = k(2)(3)$	Replace z with 18, x with 2, and y with 3.
$k = 3$	Solve for k.
$z = 3xy$	Joint variation formula with $k = 3$
$z = 3(\frac{2}{3})(\frac{5}{8}) = \frac{5}{4}$	Replace x with $\frac{2}{3}$, y with $\frac{5}{8}$ and solve for z.

Exercises - Direct, Inverse, and Joint Variations

1

Which of the following tables shows a relationship in which y is directly proportional to x?

A)

x	-2	0	2
y	-1	1	3

B)

x	-2	0	2
y	5	0	-5

C)

x	-2	0	2
y	-3	1	5

D)

x	-2	0	2
y	3	1	-1

2

The distance it takes an automobile to stop varies directly as the square of its speed. If the stopping distance of a car traveling at 40 mph is 320 feet, what is the stopping distance of a car traveling at 50mph?

A) 360 ft

B) 420 ft

C) 500 ft

D) 580 ft

3

If y varies inversely as $\sqrt{x}$, and $y = 12$ when $x = 16$, what is the value of y when $x = 100$?

A) 1.2

B) 3

C) 4.8

D) 6.4

Questions 4 and 5 refer to the following information.

$$L = \frac{k}{d^2}$$

The formula above shows the brightness of the light of an object, which varies inversely as the square of the distance. L, measured in lumens, is the brightness of the light and d, measured in meters, is the distance from the object to the light source.

4

At distance 2 meters from a light source, the brightness of an object was measured at 9 lumens. What is the value of k?

A) 18

B) 24

C) 32

D) 36

5

The brightness of an object was measured d meters away from a light source. The brightness of the same object was measured $1.5d$ meters from the light source. What is the ratio of brightness of the object when it is close to the light source to when it is farther away from the light source?

A) $\frac{9}{4}$

B) $\frac{5}{2}$

C) $\frac{7}{4}$

D) $\frac{3}{2}$

14-4. Solving Word Problems Using Rational Equations

Work Problems

You can use the following formula to solve work problems.

Work rate $\times$ Time $=$ Work done

Work rate means the amount of job done over time.

Example 1 □ Roy can finish a certain job in 12 hours and Chuck can finish the same job in 8 hours. How long will they take to finish the job together?

Solution □ Let $x =$ the number of hours needed to do the job together.

Roy's work rate is $\frac{1}{12}$ job per hour and Chuck's work rate is $\frac{1}{8}$ job per hour.

	Work rate $\times$	Time $=$	Work done
Roy	$\frac{1}{12}$	x	$\frac{1}{12}x$
Chuck	$\frac{1}{8}$	x	$\frac{1}{8}x$

Roy's part of the job $+$ Chuck's part of the job $=$ Whole job

$\frac{1}{12}x+\frac{1}{8}x=1$ Translate wording into equation.

Solving the above equation, we get $x=\frac{24}{5}$ or $4\frac{4}{5}$.

It will take them $4\frac{4}{5}$ hours to finish the job together.

Example 2 □ Pump A can fill a water tank in 6 hours and pump B can fill the same water tank in 10 hours. When the water tank was empty, both pumps were turned on for 2 hours and then pump A was turned off. How much longer did pump B have to run before the tank was filled?

Solution □ Let $x =$ the number of hours needed for pump B to fill the tank after pump A was turned off. Let $x+2 =$ total number of hours for pump B to finish the job.

Pump A's work rate is $\frac{1}{6}$ job per hour and pump B's work rate is $\frac{1}{10}$ job per hour.

	Work rate $\times$	Time $=$	Work done
Pump A	$\frac{1}{6}$	2	$\frac{1}{6}\times 2$ or $\frac{1}{3}$
Pump B	$\frac{1}{10}$	$2+x$	$\frac{1}{10}(2+x)$

Pump A's part of the job $+$ Pump B's part of the job $=$ Whole job

$\frac{1}{3}+\frac{1}{10}(2+x)=1$ Translate wording into equation.

Solving the above equation, we get $x=\frac{14}{3}$ or $4\frac{2}{3}$.

It will take $4\frac{2}{3}$ hours for pump B to finish the job.

Exercises - Solving Word Problems Using Rational Equations

Questions 1 and 2 refer to the following information.

$$\frac{1}{4}+\frac{1}{6}=\frac{1}{x}$$

Working alone, a painter can paint a house in four days. Working alone, his assistant can paint the same house in six days. Working together, they can finish painting the house in x days. The equation above represents the situation described.

1

Which of the following describes what $\frac{1}{x}$ represents in the above equation?

A) The portion of the job that the painter can finish in one day.

B) The portion of the job that the assistant can finish in one day.

C) The portion of the job that the painter and assistant together can finish in one day.

D) The portion of the job that the painter and assistant together can finish in four days.

2

How many days will it take them to finish painting the house working together?

A) $1\frac{4}{5}$

B) $2\frac{2}{5}$

C) $2\frac{4}{5}$

D) $3\frac{1}{5}$

3

Three printers A, B, and C, working together at their respective constant rates, can finish a job in 4.5 hours. Printers A and B, working together, can finish the same job in 6 hours. How many hours will it take printer C, working alone, to finish the job?

A) 12.5

B) 14

C) 16.5

D) 18

4

Mike can do a job in 48 minutes. If his brother helps him, it takes them 32 minutes. How many minutes does it take Mike's brother to do the job alone?

A) 72

B) 80

C) 96

D) 102

5

James can do a job in 8 hours and Peter can do the same job in 5 hours. If they finished $\frac{13}{25}$ of the job by working together, how long did they work together?

A) 1 hour 24 minutes

B) 1 hour 36 minutes

C) 1 hour 48 minutes

D) 2 hours 8 minutes

Chapter 14 Practice Test

1

If $a \neq b$, which of the following is equivalent to $\frac{a}{a-b}+\frac{b}{b-a}$?

A) 1

B) $\frac{a+b}{a-b}$

C) $\frac{a+b}{(a-b)^2}$

D) $\frac{a^2+b^2}{(a-b)^2}$

2

If $x>0$ and $y>0$, which of the following is equivalent to $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$?

A) $\frac{xy}{x^2-y^2}$

B) $\frac{2xy}{x^2-y^2}$

C) $\frac{xy}{x+y}$

D) $\frac{xy}{x-y}$

3

$$\frac{(k+1)^2}{k}=4k$$

What is the solution set of the equation above?

A) $\{-\frac{1}{3}\}$

B) $\{-1\}$

C) $\{-\frac{1}{3},1\}$

D) $\{\frac{1}{3},-1\}$

4

$$\frac{3}{x}-\frac{x}{x+2}=\frac{2}{x+2}$$

What is the solution set of the equation above?

A) $\{2,\ -3\}$

B) $\{-2,\ 3\}$

C) $\{-2\}$

D) $\{3\}$

5

$$\frac{x}{x+1}+\frac{4}{x-4}=\frac{20}{x^2-3x-4}$$

What is the solution set of the equation above?

A) $\{-4\}$

B) $\{4\}$

C) $\{-4,\ 4\}$

D) There are no solutions to the equation.

6

If $x \neq \pm 1$, which of the following is equivalent to $\dfrac{1+\dfrac{1}{x-1}}{1-\dfrac{1}{x+1}}$?

A) $\dfrac{x-1}{x+1}$

B) $\dfrac{x+1}{x-1}$

C) $\dfrac{x^2-1}{x^2+1}$

D) $\dfrac{x^2+1}{x^2-1}$

7

Working alone, Gary can load an empty truck in 3 hours. Working alone, his brother can load the same truck in x hours. If Gary and his brother worked together for t hours to load the empty truck, which of the following equations can be used to find out how much work was done during t hours?

A) $\dfrac{3}{t}+xt$

B) $\dfrac{3}{t}+\dfrac{x}{t}$

C) $3t+xt$

D) $\dfrac{1}{3}t+\dfrac{1}{x}t$

8

$$f(x)=\frac{5}{2(x-2)^2-3(x-2)-2}$$

What is one possible value of x, if function f is undefined?

9

If $x>0$, what is the solution to the equation $\dfrac{1}{2x}+\dfrac{3}{10x^2}=\dfrac{1}{5}$?

10

If $a \neq b$ and $\dfrac{ab}{a-b} \div \dfrac{ab^2}{b-a}=-\dfrac{1}{6}$, what is the value of b?

11

If $\dfrac{a+\dfrac{1}{2}}{a-\dfrac{1}{2}}=2$, what is the value of a?

Answer Key

Section 14-1

1. A 2. D 3. B 4. A 5. C
6. $\frac{3}{2}$

Section 14-2

1. C 2. D 3. D 4. A 5. 4
6. 9

Section 14-3

1. B 2. C 3. C 4. D 5. A

Section 14-4

1. C 2. B 3. D 4. C 5. B

Chapter 14 Practice Test

1. A 2. C 3. C 4. D 5. A
6. B 7. D 8. $\frac{3}{2}$ or 4 9. 3
10. 6 11. $\frac{3}{2}$

Answers and Explanations

Section 14-1

1. A

$$\frac{n^2}{n-4}+\frac{4n}{4-n}$$

$$=\frac{n^2}{n-4}-\frac{4n}{n-4} \qquad n-4=-(4-n)$$

$$=\frac{n^2-4n}{n-4} \qquad \text{Add the numerators.}$$

$$=\frac{n\cancel{(n-4)}}{\cancel{n-4}} \qquad \text{Factor and cancel.}$$

$$=n$$

2. D

$$\frac{a}{a^2-1}-\frac{1}{a+1}$$

$$=\frac{a}{(a+1)(a-1)}-\frac{1}{a+1} \qquad a^2-1=(a+1)(a-1)$$

$$=\frac{a}{(a+1)(a-1)}-\frac{1}{(a+1)}\cdot\frac{(a-1)}{(a-1)}$$

$$=\frac{a-(a-1)}{(a+1)(a-1)} \qquad \text{Add the numerators.}$$

$$=\frac{1}{(a+1)(a-1)}=\frac{1}{a^2-1}$$

3. B

$$\frac{y^2-1}{1+\frac{1}{y}}$$

$$=\frac{(y^2-1)y}{(1+\frac{1}{y})y} \qquad \text{Multiply the numerator and denominator by } y.$$

$$=\frac{\cancel{(y+1)}(y-1)y}{\cancel{y+1}} \qquad \text{Distributive property}$$

$$=y(y-1) \qquad \text{Simplify.}$$

4. A

$$\frac{1-\frac{1}{x+1}}{1+\frac{1}{x^2-1}}$$

$$=\frac{(1-\frac{1}{x+1})}{(1+\frac{1}{x^2-1})}\cdot\frac{(x^2-1)}{(x^2-1)} \qquad \text{Multiply } x^2-1$$

$$=\frac{(x^2-1-\frac{x^2-1}{x+1})}{(x^2-1+1)} \qquad \text{Distributive property}$$

$$=\frac{x^2-1-(x-1)}{x^2} \qquad \frac{x^2-1}{x+1}=\frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}}=x-1$$

$$=\frac{x^2-x}{x^2} \qquad \text{Simplify.}$$

$$=\frac{\cancel{x}(x-1)}{x^{\cancel{2}}}=\frac{x-1}{x} \qquad \text{Factor and cancel.}$$

5. C

$$\frac{x-3}{\frac{1}{x+2}-\frac{1}{2x-1}}$$

Multiply the numerator and the denominator by $(x+2)(2x-1)$.

$$= \frac{(x-3)[(x+2)(2x-1)]}{(\frac{1}{x+2} - \frac{1}{2x-1})[(x+2)(2x-1)]}$$

$$= \frac{(x-3)(x+2)(2x-1)}{\frac{\cancel{(x+2)}(2x-1)}{\cancel{x+2}} - \frac{(x+2)\cancel{(2x-1)}}{\cancel{2x-1}}}$$

$$= \frac{(x-3)(x+2)(2x-1)}{(2x-1)-(x+2)}$$

$$= \frac{\cancel{(x-3)}(x+2)(2x-1)}{\cancel{x-3}}$$

$$= (x+2)(2x-1)$$

6. $\frac{3}{2}$

$$\frac{x^2-xy}{2x} \div \frac{x-y}{3x^2}$$

$$= \frac{x^2-xy}{2x} \times \frac{3x^2}{x-y} \qquad \text{Rewrite as multiplication.}$$

$$= \frac{\cancel{x}\cancel{(x-y)}}{2\cancel{x}} \times \frac{3x^2}{\cancel{x-y}} \qquad \text{Factor and cancel.}$$

$$= \frac{3}{2}x^2$$

So, if $\frac{x^2-xy}{2x} \div \frac{x-y}{3x^2} = ax^2$, the value of a is $\frac{3}{2}$.

Section 14-2

1. C

$$\frac{x}{x-1} = \frac{x-2}{x+1}$$

$x(x+1) = (x-1)(x-2)$ Cross multiply.

$x^2 + x = x^2 - 3x + 2$ FOIL

$4x = 2$ Simplify.

$x = \frac{1}{2}$ Divide.

When x equals $\frac{1}{2}$, the denominator in the original equation does not have a value of 0.

The solution set is $\{\frac{1}{2}\}$.

2. D

$$\frac{x}{x-3} - 2 = \frac{4}{x-2}$$

Multiply each side by $(x-3)(x-2)$.

$$(x-3)(x-2)(\frac{x}{x-3} - 2) = (x-3)(x-2)(\frac{4}{x-2})$$

$$x(x-2) - 2(x-3)(x-2) = 4(x-3)$$

$$x^2 - 2x - 2(x^2 - 5x + 6) = 4x - 12$$

$$-x^2 + 8x - 12 = 4x - 12$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0 \text{ or } x = 4$$

When x equals 0 or 4, the denominator in the original equation does not have a value of 0.
The solution set is $\{0, 4\}$.

3. D

$$\frac{1}{x} - \frac{2}{x-2} = \frac{-4}{x^2-2x}$$

$x^2 - 2x = x(x-2)$.

So the LCD is $x(x-2)$.

Multiply each side by $x(x-2)$.

$$x(x-2)(\frac{1}{x} - \frac{2}{x-2}) = x(x-2)(\frac{-4}{x^2-2x})$$

$$(x-2) - 2x = -4$$

$$-x - 2 = -4$$

$$x = 2$$

When x equals 2, the denominator in the original equation has a value of 0.
So, the equation has no solution.

4. A

$$\frac{3}{x^2-3x} + \frac{1}{3-x} = 2$$

$$\frac{3}{x^2-3x} - \frac{1}{x-3} = 2 \qquad 3 - x = -(x-3)$$

$x^2 - 3x = x(x-3)$. So the LCD is $x(x-3)$.

Multiply each side by $x(x-3)$.

$$x(x-3)(\frac{3}{x^2-3x} - \frac{1}{x-3}) = 2x(x-3)$$

$3 - x = 2x^2 - 6x$ Distributive property

$2x^2 - 5x - 3 = 0$ Make one side 0.

$(2x+1)(x-3) = 0$ Factor.

$$x = -\frac{1}{2} \text{ or } x = 3$$

When x equals 3, the denominator in the original equation has a value of 0. Therefore, 3 cannot be a solution. The solution set is $\{-\frac{1}{2}\}$.

5. 4

If $f(x)=\dfrac{1}{(x-a)^2-4(x-a)+4}$ is undefined, the denominator $(x-a)^2-4(x-a)+4$ is equal to zero. If $x=6$, $(x-a)^2-4(x-a)+4=(6-a)^2-4(6-a)+4=0$. The expression $(6-a)^2-4(6-a)+4$ is a perfect square, which can be rewritten as $((6-a)-2)^2$. The expression $((6-a)-2)^2=0$ is equal to zero if $(6-a)-2=0$. Solving for a gives $a=4$.

6. 9

The expression $g(x)=\dfrac{1}{(x+3)^2-24(x+3)+144}$ is undefined when the denominator of $g(x)$ is zero.

$(x+3)^2-24(x+3)+144=0$

$((x+3)-12)^2=0$

$(x+3)-12=0$

$x=9$

Section 14-3

1. B

The equation of direct variation is $y=kx$, and the graph of direct variation always includes $(0,0)$. Choice B is correct.

2. C

The distance it takes an automobile to stop varies directly as the square of its speed. Thus, by the definition of direct proportionality, $d=kx^2$, in which d is the stopping distance in feet, x is the speed of the car in miles per hour, and k is a constant.

$d=kx^2$	
$320=k(40)^2$	$d=320$, $x=40$
$320=1600k$	Simplify.
$\dfrac{320}{1600}=k$	Divide each side by 1600.
$d=\dfrac{320}{1600}x^2$	Replace k with $\dfrac{320}{1600}$.
$d=\dfrac{320}{1600}(50)^2$	Substitute 50 for x.
$d=500$	

3. C

$y=\dfrac{k}{\sqrt{x}}$	Inverse variation equation
$12=\dfrac{k}{\sqrt{16}}$	$y=12$ when $x=16$.
$12=\dfrac{k}{4}$ or $k=48$	
$y=\dfrac{48}{\sqrt{x}}$	Replace k with 48.
$y=\dfrac{48}{\sqrt{100}}$	$x=100$
$y=\dfrac{48}{10}=4.8$	

4. D

$L=\dfrac{k}{d^2}$	
$9=\dfrac{k}{2^2}$	$L=9$ and $d=2$
$36=k$	

5. A

$$\frac{L \text{ measured at distance } d}{L \text{ measured at distance } 1.5d}$$

$$=\frac{\frac{36}{d^2}}{\frac{36}{(1.5d)^2}}$$

$$=\frac{36}{d^2}\cdot\frac{(1.5d)^2}{36}=\frac{2.25d^2}{d^2}$$

$$=2.25=2\frac{1}{4}=\frac{9}{4}$$

Section 14-4

1. C

Working together, they can finish painting the house in x days. So $\dfrac{1}{x}$ is the portion of the house painting job they can finish in one day.

Choice C is correct.

2. B

$$\frac{1}{4}+\frac{1}{6}=\frac{1}{x}$$

$$12x(\frac{1}{4}+\frac{1}{6})=12x(\frac{1}{x}) \qquad \text{LCD is } 12x$$
$$3x+2x=12 \qquad \text{Distributive property}$$
$$5x=12 \qquad \text{Simplify.}$$
$$x=\frac{12}{5}=2\frac{2}{5} \qquad \text{Divide each side by 5.}$$

3. D

Let a be the number of days it takes printer A to finish the job alone, let b be the number of days it takes printer B to finish the job alone, and let c be the number of days it takes printer C to finish the job alone. Then their respective work rates are $\frac{1}{a}$, $\frac{1}{b}$, and $\frac{1}{c}$. If three printers A, B, and C, working together at their respective constant rates, can finish a job in $4\frac{1}{2}$ hours, you can set up the equation $4\frac{1}{2}(\frac{1}{a}+\frac{1}{b}+\frac{1}{c})=1$. If printers, A and B, working together at their respective constant rates, can finish a job in 6 hours, you can set up the equation $6(\frac{1}{a}+\frac{1}{b})=1$.

Solving the two equations for $\frac{1}{a}+\frac{1}{b}$ gives

$$4\frac{1}{2}(\frac{1}{a}+\frac{1}{b}+\frac{1}{c})=1 \;\Rightarrow\; \frac{9}{2}(\frac{1}{a}+\frac{1}{b}+\frac{1}{c})=1$$
$$\Rightarrow\; \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{2}{9} \;\Rightarrow\; \frac{1}{a}+\frac{1}{b}=\frac{2}{9}-\frac{1}{c} \text{ and}$$
$$6(\frac{1}{a}+\frac{1}{b})=1 \;\Rightarrow\; \frac{1}{a}+\frac{1}{b}=\frac{1}{6}.$$

Substituting $\frac{2}{9}-\frac{1}{c}$ for $\frac{1}{a}+\frac{1}{b}$ gives $\frac{2}{9}-\frac{1}{c}=\frac{1}{6}$. Multiply by $18c$ on each side of the equation and simplify.

$$18c(\frac{2}{9}-\frac{1}{c})=18c(\frac{1}{6})$$
$$4c-18=3c$$
$$c=18$$

4. C

Let b be the number of minutes for his brother to do the job alone. Since the part of the job Mike does in 32 minutes plus the part of the job his brother does in 32 minutes equals one whole job, you can set up the following equation.

$$32(\frac{1}{48}+\frac{1}{b})=1$$
$$48b\cdot 32(\frac{1}{48}+\frac{1}{b})=48b\cdot 1 \qquad \text{LCD is } 48b.$$
$$32b+1536=48b \qquad \text{Simplify.}$$
$$16b=1536$$
$$b=96$$

5. B

If James can do a job in 8 hours, his work rate is $\frac{1}{8}$. If Peter can do the same job in 5 hours, his work rate is $\frac{1}{5}$.

Let $x=$ the number of hours they worked together.

$$\frac{1}{8}x+\frac{1}{5}x=\frac{13}{25}$$
$$200(\frac{1}{8}x+\frac{1}{5}x)=200\cdot\frac{13}{25} \qquad \text{LCD is } 200.$$
$$25x+40x=104 \qquad \text{Simplify.}$$
$$65x=104$$
$$x=\frac{104}{65}=1.6$$

0.6 hours is 0.6×60 minutes, or 36 minutes. Therefore, it took 1 hour and 36 minutes for them to finish $\frac{13}{25}$ of the job.

Chapter 14 Practice Test

1. A

$$\frac{a}{a-b}+\frac{b}{b-a}$$
$$=\frac{a}{a-b}-\frac{b}{a-b} \qquad b-a=-(a-b)$$
$$=\frac{a-b}{a-b} \qquad \text{Add the numerators.}$$
$$=1$$

2. C

$$\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}}$$

Multiply x^2y^2 by the numerator and the denominator.

$$= \frac{(\frac{1}{x}-\frac{1}{y})x^2y^2}{(\frac{1}{x^2}-\frac{1}{y^2})x^2y^2}$$

$$= \frac{xy^2 - x^2y}{y^2 - x^2} \qquad \text{Distributive property}$$

$$= \frac{xy\,\cancel{(y-x)}}{\cancel{(y-x)}\,(y+x)}$$

$$= \frac{xy}{(y+x)}$$

3. C

$$\frac{(k+1)^2}{k} = 4k$$

$(k+1)^2 = 4k^2$ Multiply by k on each side.

$k^2 + 2k + 1 = 4k^2$ FOIL

$0 = 3k^2 - 2k - 1$ Make one side 0.

$0 = (3k+1)(k-1)$ Factor.

$k = -\frac{1}{3}$ or $k = 1$

None of the solutions make the denominator zero, thus $\{-\frac{1}{3}, 1\}$ is the solution set.

Choice C is correct.

4. D

$$\frac{3}{x} - \frac{x}{x+2} = \frac{2}{x+2}$$

Multiply each side of the equation by $x(x+2)$.

$$x(x+2)(\frac{3}{x} - \frac{x}{x+2}) = x(x+2)(\frac{2}{x+2})$$

$3(x+2) - x^2 = 2x$ Distributive property

$3x + 6 - x^2 = 2x$ Distributive property

$0 = x^2 - x - 6$ Make one side 0.

$0 = (x+2)(x-3)$ Factor.

$x = -2$ or $x = 3$

When x equals -2, the denominator in the original equation has a value of 0. Therefore, -2 cannot be a solution.

The solution set is $\{3\}$.

5. A

$$\frac{x}{x+1} + \frac{4}{x-4} = \frac{20}{x^2 - 3x - 4}$$

$x^2 - 3x - 4 = (x+1)(x-4)$. So the LCD is $(x+1)(x-4)$. Multiply each side of the equation by $(x+1)(x-4)$.

$$(x+1)(x-4)(\frac{x}{x+1} + \frac{4}{x-4})$$

$$= (x+1)(x-4)(\frac{20}{x^2-3x-4})$$

$x(x-4) + 4(x+1) = 20$ Distributive property

$x^2 - 4x + 4x + 4 = 20$

$x^2 = 16$

$x = 4$ or $x = -4$

When x equals 4, the denominator in the original equation has a value of 0. Therefore, 4 cannot be a solution.
The solution set is $\{-4\}$.

6. B

$$\frac{1 + \frac{1}{x-1}}{1 - \frac{1}{x+1}}$$

$$= \frac{(x+1)(x-1)(1+\frac{1}{x-1})}{(x+1)(x-1)(1-\frac{1}{x+1})} \qquad \text{Multiply by } (x+1)(x-1).$$

$$= \frac{(x+1)(x-1) + (x+1)}{(x+1)(x-1) - (x-1)} \qquad \text{Distributive property}$$

$$= \frac{x^2 - 1 + x + 1}{x^2 - 1 - x + 1} \qquad \text{FOIL}$$

$$= \frac{x^2 + x}{x^2 - x} \qquad \text{Simplify.}$$

$$= \frac{x(x+1)}{x(x-1)} \qquad \text{Factor.}$$

$$= \frac{x+1}{x-1} \qquad \text{Cancel and simplify.}$$

7. D

If working alone Gary can load the empty truck in 3 hours, his work rate is $\frac{1}{3}$. If working alone his brother can load the same truck in x hours, his work rate is $\frac{1}{x}$. If they work together for t

hours to load the empty truck, the amount of work done for t hours will be $t(\frac{1}{3}+\frac{1}{x})$, or $\frac{1}{3}t+\frac{1}{x}t$.

8. $\frac{3}{2}$ or 4

The expression $f(x)=\dfrac{5}{2(x-2)^2-3(x-2)-2}$ is undefined when the denominator of $f(x)$ is zero. Therefore, if $2(x-2)^2-3(x-2)-2$ is equal to 0, $f(x)$ is undefined.

$2(x-2)^2-3(x-2)-2=0$

Let $z=x-2$, then $2z^2-3z-2=0$.

$(2z+1)(z-2)=0$ Factor.

$2z+1=0$ or $z-2=0$ Zero Product Property

$z=-\frac{1}{2}$ or $z=2$

Now substitute $x-2$ for z.

$x-2=-\frac{1}{2}$ or $x-2=2$

The values of x that make f undefined are $\frac{3}{2}$ and 4.

9. 3

$$\frac{1}{2x}+\frac{3}{10x^2}=\frac{1}{5}$$

Multiply each side of the equation by $10x^2$.

$10x^2(\frac{1}{2x}+\frac{3}{10x^2})=10x^2(\frac{1}{5})$

$5x+3=2x^2$ Distributive property

$0=2x^2-5x-3$ Make one side 0.

$0=(2x+1)(x-3)$ Factor.

$x=-\frac{1}{2}$ or $x=3$

Since $x>0$, the only solution is 3.

10. 6

$$\frac{ab}{a-b}\div\frac{ab^2}{b-a}=-\frac{1}{6}$$

Rewrite as multiplication.

$$\frac{ab}{a-b}\times\frac{b-a}{ab^2}=-\frac{1}{6}$$

$$\frac{\cancel{ab}}{\cancel{a-b}}\times\frac{-\cancel{(a-b)}}{\cancel{ab}^2}=-\frac{1}{6}$$ $b-a=-(a-b)$

$$\frac{-1}{b}=-\frac{1}{6}$$

Therefore, the value of b is 6.

11. $\frac{3}{2}$

$$\frac{a+\frac{1}{2}}{a-\frac{1}{2}}=2$$

Multiply each side of the equation by $a-\frac{1}{2}$.

$a+\frac{1}{2}=2(a-\frac{1}{2})$

$a+\frac{1}{2}=2a-1$ Distributive property

$\frac{3}{2}=a$