Answer Key

Section 12-1

1. B 2. C 3. A 4. D 5. 6
6. 4

Section 12-2

1. B 2. B 3. D 4. C 5. A

Section 12-3

1. A 2. D 3. 395

Section 12-4

1. D 2. B 3. C 4. B 5. 1.04
6. 446

Chapter 12 Practice Test

1. B 2. A 3.C 4. D 5. B
6. C 7. 8485 8. 10 9. 283

Answers and Explanations

Section 12-1

1. B

$g(x) = 1 - x$

$g(-2) = 1 - (-2)$ Substitute -2 for x.

$= 3$

$f(x) = x^2 - 3x - 1$

$f \circ g(-2) = f(g(-2))$

$= f(3)$ $g(-2) = 3$

$= (3)^2 - 3(3) - 1$ Substitute 3 for x.

$= -1$

2. C

$f = \{(-4,12), (-2,4), (2,0), (3,\frac{3}{2})\} \Rightarrow$

$f(-4) = 12$, $f(-2) = 4$, $f(2) = 0$ and $f(3) = \frac{3}{2}$

$g = \{(-2,5), (0,1), (4,-7), (5,-9)\} \Rightarrow$

$g(-2) = 5$, $g(0) = 1$, $g(4) = -7$, $g(5) = -9$

$g \circ f(2) = g(f(2))$

$= g(0)$ $f(2) = 0$

$= 1$ $g(0) = 1$

3. A

$f(g(-1))$

$= f(1)$ $g(-1) = 1$

$= -2$ $f(1) = -2$

4. D

$g(x) = 2 - x$

$g(3) = 2 - 3$ Substitute 3 for x.

$= -1$

$f(g(3))$

$= f(-1)$ $g(3) = -1$

$= \frac{1-5(-1)}{2}$ Substitute -1 for x.

$= 3$

5. 6

x	$f(x)$	$g(x)$
-2	-5	0
0	6	4
3	0	-5

Based on the table, $g(-2) = 0$.

$f(g(-2))$

$= f(0)$ $g(-2) = 0$

$= 6$

6. 4

Based on the table, $f(3) = 0$.

$g(f(3))$

$= g(0)$ $f(3) = 0$

$= 4$

Section 12-2

1. B

$a_n = \sqrt{(a_{n-1})^2 + 2}$

$a_1 = \sqrt{(a_0)^2 + 2}$ $n = 1$

$= \sqrt{(\sqrt{2})^2 + 2}$ $a_0 = \sqrt{2}$

$= \sqrt{4} = 2$

$a_2 = \sqrt{(a_1)^2 + 2}$ $\quad n = 2$

$= \sqrt{(2)^2 + 2}$ $\quad a_1 = 2$

$= \sqrt{6}$

2. B

$a_{n+1} = a_n - \dfrac{f(a_n)}{g(a_n)}$

$a_1 = a_0 - \dfrac{f(a_0)}{g(a_0)}$ $\quad n = 0$

$= 1 - \dfrac{f(1)}{g(1)}$ $\quad a_0 = 1$

Since $f(x) = x^2 - 3x$ and $g(x) = 2x - 3$,
$f(1) = (1)^2 - 3(1) = -2$ and $g(1) = 2(1) - 3 = -1$.

Thus, $a_1 = 1 - \dfrac{f(1)}{g(1)} = 1 - \dfrac{-2}{-1} = -1$.

$a_2 = a_1 - \dfrac{f(a_1)}{g(a_1)}$ $\quad n = 1$

$= -1 - \dfrac{f(-1)}{g(-1)}$ $\quad a_1 = -1$

$f(-1) = (-1)^2 - 3(-1) = 4$ and
$g(-1) = 2(-1) - 3 = -5$.

Thus, $a_2 = -1 - \dfrac{f(-1)}{g(-1)} = -1 - \dfrac{4}{-5} = -\dfrac{1}{5}$

3. D

$f(x) = \sqrt{2x^2 - 1}$

$f \circ f \circ f(2)$

$= f(f(f(2)) = f(f(\sqrt{2(2)^2 - 1}))$

$= f(f(\sqrt{7})) = f(\sqrt{2(\sqrt{7})^2 - 1})$

$= f(\sqrt{13}) = \sqrt{2(\sqrt{13})^2 - 1}$

$= \sqrt{25} = 5$

4. C

$A_n = (1 + \dfrac{r}{100}) \cdot A_{n-1} + 12b$

$A_1 = (1 + \dfrac{r}{100}) \cdot A_0 + 12b$ $\quad n = 1$

$= (1 + \dfrac{5}{100}) \cdot 12,000 + 12(400)$

$= 17,400$

$A_2 = (1 + \dfrac{r}{100}) \cdot A_1 + 12b$ $\quad n = 2$

$= (1 + \dfrac{5}{100}) \cdot 17,400 + 12(400)$ $\quad A_1 = 17,400$

$= 23,070$

$A_3 = (1 + \dfrac{r}{100}) \cdot A_2 + 12b$ $\quad n = 3$

$= (1 + \dfrac{5}{100}) \cdot 23,070 + 12(400)$ $\quad A_2 = 23,070$

$= 29,023.50$

5. A

$P_n = 0.85P_{n-1} + 20$

$P_1 = 0.85P_0 + 20$ $\quad n = 1$

$= 0.85(400) + 20$ $\quad P_0 = 400$

$= 360$

$P_2 = 0.85P_1 + 20$ $\quad n = 2$

$= 0.85(360) + 20$ $\quad P_1 = 360$

$= 326$

$P_3 = 0.85P_2 + 20$ $\quad n = 2$

$= 0.85(326) + 20$ $\quad P_2 = 326$

$= 297.1$

Section 12-3

1. A

Suppose the initial water level was 100 units. If the water level decreases by 10 percent each year, the water level will be $100(1 - 0.1)^n$, or $100(0.9)^n$, n years later. The water level decreases exponentially, not linearly. Of the graphs shown, only choice A would appropriately model exponential decrease.

2. D

I. At time $t = 0$, the price of model A was \$30,000 and the price of model B was \$24,000. To find out what percent the price of model A was higher than the price of model B, use the following equation.

$30,000 = 24,000(1 \underbrace{+ \dfrac{x}{100}}_{x\% \text{ more than}})$

$\dfrac{30,000}{24,000} = 1 + \dfrac{x}{100}$

$\Rightarrow 1.25 = 1 + \dfrac{x}{100} \Rightarrow 0.25 = \dfrac{x}{100}$

$\Rightarrow 25 = x$

Therefore the price of model A was 25% higher than and the price of model B.

Roman numeral I is true.

To find out what percent the price of model B was less than the price of model A, use the following equation.

$$24,000 = 30,000(1 \underbrace{- \frac{x}{100}}_{x\% \text{ less than}})$$

$$\frac{24,000}{30,000} = 1 - \frac{x}{100}$$

$$0.8 = 1 - \frac{x}{100} \Rightarrow 0.2 = \frac{x}{100}$$

$$\Rightarrow 20 = x$$

Therefore the price of model B was 20% less than the price of model A.

Roman numeral II is true.

From time $t = 0$ to $t = 6$, the average rate of decrease in the value of model A

$$= \frac{\text{amount of decrease}}{\text{change in years}} = \frac{30,000 - 12,000}{6}$$

$$= 3,000$$

From time $t = 0$ to $t = 6$, the average rate of decrease in the value of model B

$$= \frac{\text{amount of decrease}}{\text{change in years}} = \frac{24,000 - 12,000}{6}$$

$$= 2,000$$

Therefore, from time $t = 0$ to $t = 6$, the average rate of decrease in the value of model A was 1.5 times the average rate of decrease in the value of model B.

Roman numeral III is also true.

Choice D is correct.

3. 395

$f(x) = 12,000(0.9)^x$ and $g(x) = 14,000(0.85)^x$

$g(2) - f(2) = 14,000(0.85)^2 - 12,000(0.9)^2$

$= 10,115 - 9720 = 395$

Section 12-4

1. D

The present population must be multiplied by a factor of 2 to double. If a certain population doubles every 40 days, the population grows by a multiple of $(2)^{\frac{1}{40}}$ each day. After t days, the population will be multiplied by $(2)^{\frac{t}{40}}$. If the population starts with 12 rabbits, after t days, the population will be $12 \times (2)^{\frac{t}{40}}$.

2. B

For the present population to decrease by 4%, the initial population must be multiplied by a factor of 0.96. If population P is 80,000 this year, it will be
$80,000(0.96)$ one year later,
$80,000(0.96)(0.96)$ two years later,
$80,000(0.96)(0.96)(0.96)$ three years later,
and so on. After t years, the population will be $80,000(0.96)^t$.

3. C

For the price of a house to increase at an annual growth rate of r, it must be multiplied by a factor of $(1+r)$ each year. If the price of the house is \$150,000 this year, it will be
$150,000(1+r)$ one year later,
$150,000(1+r)(1+r)$ two years later,
$150,000(1+r)(1+r)(1+r)$ three years later,
and so on. Thus, 10 years later, the price of the house will be $150,000(1+r)^{10}$.

4. B

If the half-life of a substance is 12 days, half of the substance decays every 12 days.
Make a chart.

Amount	Days
128	0
$128 \times \frac{1}{2}$	12 days after
$128 \times \frac{1}{2} \times \frac{1}{2}$	24 days after
$128 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$	36 days after
$128 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$	48 days after

Therefore, after 48 days, there will be $128 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, or 8 milligrams, of the radioactive substance left.

5. 1.04

The initial deposit earns 4 percent interest compounded annually. Thus at the end of one year, the new value of the account is the initial deposit of \$3,000 plus 4 percent of the initial deposit: $\$3,000 + 0.04(\$3,000) = \$3,000(1+0.04)$.
Since the interest is compounded annually, the value at the end of each succeeding year is the previous year's value plus 4 percent of the previous year's value. Thus after 2 years, the value will be $\$3,000(1.04)(1.04)$. After 3 years, the value will be $\$3,000(1.04)(1.04)(1.04)$.

After t years, the value will be $\$3,000(1.04)^t$.

Therefore, the value of x in the expression $\$3,000(x)^t$ is 1.04.

6. 446

The difference in the amount after 10 years will be
$\$3,000(1.05)^{10} - \$3,000(1.04)^{10}$
$\approx \$445.95$.
To the nearest dollar the difference in the amount will be \$446.

Chapter 12 Practice Test

1. B

$f(x) = \sqrt{2x}$ and $g(x) = 2x^2$

$g(1) = 2(1)^2 = 2$ and $f(1) = \sqrt{2(1)} = \sqrt{2}$

$f(g(1)) - g(f(1))$

$= f(2) - g(\sqrt{2})$

$= \sqrt{2(2)} - 2(\sqrt{2})^2$

$= \sqrt{4} - 2(2) = 2 - 4 = -2$

2. A

$f(x) = \sqrt{625 - x^2}$ and $g(x) = \sqrt{225 - x^2}$

$f(5) = \sqrt{625 - 5^2} = \sqrt{600}$

$g(5) = \sqrt{225 - 5^2} = \sqrt{200}$

$f(f(5)) - g((g5))$

$= f(\sqrt{600}) - g(\sqrt{200})$

$= (\sqrt{625 - (\sqrt{600})^2}) - (\sqrt{225 - (\sqrt{200})^2})$

$= \sqrt{625 - 600} - \sqrt{225 - 200}$

$= \sqrt{25} - \sqrt{25} = 0$

3. C

Method I:

You can keep dividing by 2 until you get to a population of 6,400.

Year	Population
1980	51,200
1955	25,600
1930	12,800
1905	6,400

Method II:

Use the half-life formula, $A = P(\frac{1}{2})^{t/d}$.

$6,400 = 51,200(\frac{1}{2})^{t/25}$

$\frac{6,400}{51,200} = (\frac{1}{2})^{t/25}$ Divide each side by 51,200.

$\frac{1}{8} = (\frac{1}{2})^{t/25}$ Simplify.

$(\frac{1}{2})^3 = (\frac{1}{2})^{t/25}$ $\frac{1}{8} = (\frac{1}{2})^3$

$3 = \frac{t}{25}$ If $b^x = b^y$, then $x = y$.

$75 = t$

Therefore, in year $1980 - 75$, or 1905, the population of the town was 6,400.

4. D

The table shows that one-half of the substance decays every 28 years. Therefore, the half-life of the radioactive substance is 28 years. Use the half-life formula, $A = P(\frac{1}{2})^{t/d}$, to find out how much of the original amount of the substance will remain after 140 years. P is the initial amount, t is the number of years and d is the half-life.

$A = 1,200(\frac{1}{2})^{140/28}$

$= 37.5$ Use a calculator.

To the nearest gram, 38 grams of the substance will remain after 140 years.

5. B

If the substance decays at a rate of 18% per year the amount of substance remaining each year will be multiplied by $(1-0.18)$, or 0.82.
The initial amount of 100 grams will become

$100(1-0.18)$ one year later,

$100(1-0.18)(1-0.18)$ two years later,

$100(1-0.18)(1-0.18)(1-0.18)$ three years later,

and so on. Thus, t years later, the remaining amount of the substance, in grams, is $f(t)=100(0.82)^t$.

6. C

$$5,000(1+\frac{r}{100})^t$$

The value of the 15 year investment at 6% annual compound interest

$$=5,000(1+\frac{6}{100})^{15}=5,000(1.06)^{15}.$$

The value of the 12 year investment at 6% annual compound interest

$$=5,000(1+\frac{6}{100})^{12}=5,000(1.06)^{12}.$$

The difference is

$$=5,000(1.06)^{15}-5,000(1.06)^{12}$$

$$=5,000\left[(1.06)^{15}-(1.06)^{12}\right]$$

7. 8485

$$P(t)=24,000(\frac{1}{2})^{\frac{t}{6}}$$

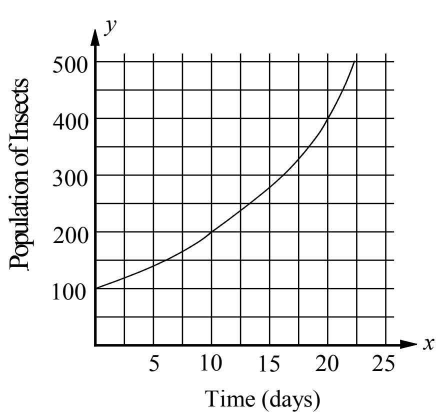
$$P(9)=24,000(\frac{1}{2})^{\frac{9}{6}} \qquad \text{Substitute 9 for } t.$$

$$=24,000(\frac{1}{2})^{\frac{3}{2}}$$

$$\approx 8,485.28 \qquad \text{Use a calculator.}$$

To the nearest dollar, the price of the truck 9 years after it was purchased is \$8,485.

8. 10



$$f(t)=100(2)^{\frac{t}{d}}$$

In the equation, d represents the amount of time it takes to double the population. The graph shows that the population was 100 at $t=0$, 200 at $t=10$, and 400 at $t=20$. Therefore, the value of doubling time d is 10 days.

9. 283

$$f(t)=100(2)^{\frac{t}{d}}$$

$$f(15)=100(2)^{\frac{15}{10}}=100(2)^{1.5}$$

$$\approx 282.84 \qquad \text{Use a calculator.}$$

The population of the insect after 15 days was 283, to the nearest whole number.