CHAPTER 10
Laws of Exponents and Polynomials

10-1. Laws of Exponents and Scientific Notation

Laws of Exponents

For all integers m and n and any nonzero numbers a and b, the following properties hold.

Symbols	Example
$a^m \cdot a^n = a^{m+n}$	$2^3 \cdot 2^5 = 2^{3+5} = 2^8$
$(a^m)^n = a^{m \cdot n}$	$(2^3)^5 = 2^{3 \cdot 5} = 2^{15}$
$(ab)^m = a^m b^m$	$(-2x)^5 = (-2)^5 x^5 = -32x^5$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^5}{2^3} = 2^{5-3} = 2^2$
$(\frac{a}{b})^m = \frac{a^m}{b^m}$	$(\frac{2x}{5})^3 = \frac{(2x)^3}{5^3} = \frac{8x^3}{125}$
$a^0 = 1$	$(-2xy)^0 = 1$
$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$	$5^{-2} = \frac{1}{5^2}$ and $\frac{1}{4^{-3}} = 4^3$
$(\frac{a}{b})^{-m} = (\frac{b}{a})^m = \frac{b^m}{a^m}$	$(\frac{-2x}{3y^3})^{-2} = (\frac{3y^3}{-2x})^2 = \frac{(3y^3)^2}{(-2x)^2} = \frac{9y^6}{4x^2}$

Example 1 □ Simplify.

a. $(-2ab^2)(3a^5b^3) =$

b. $[(a^2)^3]^4 =$

c. $x^{n-2} \cdot x \cdot x^{n+1}$

d. $\frac{a^7b^4c^2}{a^5b^2c} =$

e. $(\frac{3a^2bc^3}{2ab^2})^3 =$

f. $(\frac{7a^{-1}bc^3}{4a^2b^4})^0 =$

g. $\frac{a^{-2}b^3c^{-3}}{ab^{-2}} =$

h. $(\frac{2p^2}{3q})^{-3} =$

Solution □ a. $(-2ab^2)(3a^5b^3) = (-2)(3)(a \cdot a^5)(b^2 \cdot b^3)$ Group the coefficients and variables.

$= -6a^6b^5$ $\quad a^m \cdot a^n = a^{m+n}$

b. $[(a^2)^3]^4 = [a^6]^4 = a^{24}$ $\quad (a^m)^n = a^{m \cdot n}$

c. $x^{n-2} \cdot x \cdot x^{n+1} = x^{(n-2)+1+(n+1)}$ $\quad a^m \cdot a^n = a^{m+n}$

$= x^{2n}$ Simplify.

d. $\frac{a^7b^4c^2}{a^5b^2c} = (\frac{a^7}{a^5})(\frac{b^4}{b^2})(\frac{c^2}{c})$ Group powers that have the same base.

$= a^2b^2c$ $\quad \frac{a^m}{a^n} = a^{m-n}$

e. $(\frac{3a^2bc^3}{2ab^2})^3 = \frac{(3a^2bc^3)^3}{(2ab^2)^3}$ — $(\frac{a}{b})^m = \frac{a^m}{b^m}$

$= \frac{3^3(a^2)^3(b)^3(c^3)^3}{(2)^3(a)^3(b^2)^3}$ — $(ab)^m = a^m b^m$

$= \frac{27a^6b^3c^9}{8a^3b^6}$ — $(a^m)^n = a^{m\cdot n}$

$= \frac{27a^3c^9}{8b^3}$ — $\frac{a^m}{a^n} = a^{m-n}$

f. $(\frac{7a^{-1}bc^3}{4a^2b^4})^0 = 1$ — $a^0 = 1$

g. $\frac{a^{-2}b^3c^{-3}}{ab^{-2}} = \frac{b^3b^2}{aa^2c^3}$ — $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

$= \frac{b^5}{a^3c^3}$ — Simplify.

h. $(\frac{2p^2}{3q})^{-3} = (\frac{3q}{2p^2})^3 = \frac{(3q)^3}{(2p^2)^3}$ — $(\frac{a}{b})^{-m} = (\frac{b}{a})^m = \frac{b^m}{a^m}$

$= \frac{27q^3}{8p^6}$ — Simplify.

Scientific Notation

A number is in **scientific notation** when it is in the form $a \times 10^n$, in which $1 \le a < 10$ and n are integers.

Example 2 □ Write each number in scientific notation.

a. $205{,}000$

b. 0.000107

Simplify and write each number in decimal form.

c. $(5\times 10^{-12})(3\times 10^4)$

d. $\frac{6\times 10^3}{12\times 10^{-4}}$

Solution □

a. $205{,}000 = 2.05\times 10^5$ — Decimal point moved 5 places to the left.

b. $0.000107 = 1.07\times 10^{-4}$ — Decimal point moved 4 places to the right.

c. $(5\times 10^{-12})(3\times 10^4) = 15\times 10^{-8}$ — $10^{-12}\cdot 10^4 = 10^{-8}$

$= 1.5\times 10\times 10^{-8} = 1.5\times 10^{-7}$ — Simplify.

d. $\frac{6\times 10^3}{12\times 10^{-4}} = \frac{6}{12}\times\frac{10^3}{10^{-4}}$ — Group the coefficients and variables.

$= \frac{1}{2}\times 10^7$ — $\frac{10^3}{10^{-4}} = 10^{3-(-4)} = 10^7$

$= 0.5\times 10^7 = 5\times 10^{-1}\times 10^7 = 5\times 10^6$ — Simplify.

Exercises - Laws of Exponents and Scientific Notation

1

If $(-a^2b^3)(2ab^2)(-3b) = ka^m b^n$, what is the value of $m+n$?

2

If $(\frac{2}{3}a^2b)^2(\frac{4}{3}ab)^{-3} = ka^m b^n$, what is the value of k?

3

If $\frac{(x)^3(-y)^2 z^{-2}}{(x)^{-2} y^3 z} = \frac{x^m}{y^n z^p}$, what is the value of $m+n+p$?

4

If $2^x = 5$, what is the value of $2^x + 2^{2x} + 2^{3x}$?

5

$$(3^x + 3^x + 3^x)\cdot 3^x$$

Which of the following is equivalent to the expression shown above?

A) 3^{4x}

B) 3^{3x^2}

C) 3^{1+3x}

D) 3^{1+2x}

6

$$\frac{(6xy^2)(2xy)^2}{8x^2y^2}$$

If the expression above is written in the form $ax^m y^n$, what is the value of $m+n$?

7

If x is not equal to zero, what is the value of $\frac{(2x)^3(3x)}{(6x^2)^2}$?

8

If $8,200\times 300,000$ is equal to 2.46×10^n, what is the value of n?

9

If $\frac{240}{80,000}\times\frac{6,000}{900,000}$ is equal to $\frac{1}{5\times 10^n}$, what is the value of n?

10-2. Adding, Subtracting, Multiplying, and Dividing Polynomials

An expression with the form ax^n, in which n is a nonnegative integer, is called a **polynomial**. Polynomials that have only one term are called **monomials**. Polynomials that have two unlike terms are called **binomials**, and that have three unlike terms are called **trinomials**.
Terms such as $3x^2$ and $-2x^2$ are called **like terms** because they have the same variable to the same power.

The **degree of monomial** is the sum of the exponents of all of the monomial's variables.
The **degree of polynomial** is the greatest degree of any term in the polynomial.

Example 1 □ Find the degree of the polynomial $x-3xy-12+5x^2y$.

Solution □ Degree of x, $-3xy$, -12, and $5x^2y$ are 1, $1+1$, 0, and $2+1$ respectively.

Therefore the degree of the polynomial is 3.

Adding and Subtracting Polynomials
To add or subtract two polynomials, add or subtract the coefficient of like terms.

Example 2 □ Find each sum or difference.

a. $(5x^2-3x+14)+(2x^2+x-9)$. b. $(-2p^3+7p^2-1)-(4p^3+p-6)$

Solution □ a. $(5x^2-3x+14)+(2x^2+x-9)$

$=(5x^2+2x^2)+(-3x+x)+(14+(-9))$

$=7x^2-2x+5$

b. $(-2p^3+7p^2-1)-(4p^3+p-6)$

$=-2p^3+7p^2-1-4p^3-p+6$

$=(-2p^3-4p^3)+(7p^2)+(-p)+(-1+6)$

$=-6p^3+7p^2-p+5$

Multiplying Polynomials
The Distributive Property and the various laws of exponents can be used to multiply a polynomial by a monomial.

Example 3 □ Simplify each expression.

a. $-3x^2(2x^2-3x+5)$ b. $5(a^2+3)-a(a^2+7a-2)$

Solution □ a. $-3x^2(2x^2-3x+5)$

$=-3x^2(2x^2)-(-3x^2)(3x)-3x^2(5)$ Distributive Property

$=-6x^4+9x^3-15x^2$ Simplify.

b. $5(a^2+3)-a(a^2+7a-2)$

$=5(a^2)+5(3)-a(a^2)-a(7a)-a(-2)$ Distributive Property

$=5a^2+15-a^3-7a^2+2a$ Multiply.

$=-a^3-2a^2+2a+15$ Combine like terms

Dividing Polynomials

To divide a polynomial by a monomial, divide each term by the monomial, add the results, and simplify. When dividing a polynomial function $f(x)$ by $ax+b$, **long division,** which is similar to long division in arithmetic, can be used to find quotient $q(x)$ and remainder R.

The division can be written as $\dfrac{f(x)}{ax+b}=q(x)+\dfrac{R}{ax+b}$.

Example 4 □ Divide.

a. $(3x^2-8x+4)\div(2x)$

b. $(x^3+2x^2-5x+9)\div(x-2)$

c. $(2x^3-15x+9)\div(x+3)$

Solution □ a. $(3x^2-8x+4)\div(2x)$

$=\dfrac{3x^2-8x+4}{2x}$	Write in fraction form.
$=\dfrac{3x^2}{2x}-\dfrac{8x}{2x}+\dfrac{4}{2x}$	Divide each term in $3x^2-8x+4$ by $2x$.
$=\dfrac{3x}{2}-4+\dfrac{2}{x}$	Simplify.

b.

x^2+4x+3	Quotient
Divisor ⟶ $x-2\,\overline{)\,x^3+2x^2-5x+9}$	Dividend
$\underline{x^3-2x^2}$	$x^2\times(x-2)=x^3-2x^2$
$4x^2-5x$	Result of subtraction
$\underline{4x^2-8x}$	$4x\times(x-2)=4x^2-8x$
$3x+9$	Result of subtraction
$\underline{3x-6}$	$3\times(x-2)=3x-6$
15	The remainder is 15.

$$(x^3+2x^2-5x+9)\div(x-2)=x^2+4x+3+\frac{15}{x-2}.$$

c.

$2x^2-6x+3$	Quotient
Divisor ⟶ $x+3\,\overline{)\,2x^3+0x^2-15x+9}$	Write $0x^2$.
$\underline{2x^3+6x^2}$	$2x^2\times(x+3)=2x^3+6x^2$
$-6x^2-15x$	Result of subtraction
$\underline{-6x^2-18x}$	$-6x\times(x+3)=-6x^2-18x$
$3x+9$	Result of subtraction
$\underline{3x+9}$	$3\times(x+3)=3x+9$
0	The remainder is 0.

$$(2x^3-15x+9)\div(x+3)=2x^2-6x+3$$

Exercises – Adding, Subtracting, Multiplying, and Dividing Polynomials

1

$$a(2-a)+(a^2+3)-(2a+1)$$

Which of the following is equivalent to the expression shown above?

A) 2

B) $4a$

C) $2a+2$

D) $2a-2$

2

$$(-m^2n-n^2+3mn^2)-(m^2n-n^2+mn^2)$$

Which of the following is equivalent to the expression shown above?

A) $4mn^2$

B) $4m^2n$

C) $-2m^2n+2mn^2$

D) $2m^2n+2mn^2$

3

$$(2x^2-3x+1)-(-2x^2-3x+2)$$

If the expression above is written in the form ax^2+bx+c, in which a, b, and c are constants, what is the value of $a+b+c$?

A) 2

B) 3

C) 4

D) 5

4

$$(x^3-x^2+3x-3)\div(x-1)$$

Which of the following is the quotient of the expression shown above?

A) x^2-3

B) x^2+3

C) x^2-2x

D) x^2-2x+3

5

$$(14x^2+9x-20)\div(ax-1)=7x+8+\frac{-12}{ax-1}$$

In the equation above, a is a constant and $ax-1\neq 0$. What is the value of a?

6

If $\dfrac{6x^2-5x+4}{-3x+1}=-2x+1+\dfrac{A}{-3x+1}$, what is the value of A?

10-3. FOIL Method and Special Products

FOIL Method

The product of two binomials is the sum of the products of the first terms, the outer terms, the inner terms, and the last terms.

$$\underbrace{(2x+3)(x-5)}_{} = \overset{\text{Product of First Terms}}{(2x)(x)} + \overset{\text{Product of Outer Terms}}{(2x)(-5)} + \overset{\text{Product of Inner Terms}}{(3)(x)} + \overset{\text{Product of Last Terms}}{(3)(-5)}$$

$$= 2x^2 - 10x + 3x - 15$$

$$= 2x^2 - 7x - 15$$

Example 1 □ Simplify.

a. $(x+3)(2x-9)$ b. $(3n-2)(2n^2-n+5)$

Solution □ a. $(x+3)(2x-9) = x(2x) + x(-9) + 3(2x) + 3(-9)$

$$= 2x^2 - 9x + 6x - 27 = 2x^2 - 3x - 27$$

b. $(3n-2)(2n^2-n+5) = 3n(2n^2-n+5) - 2(2n^2-n+5)$

$$= (6n^3 - 3n^2 + 15n) + (-4n^2 + 2n - 10)$$

$$= 6n^3 - 7n^2 + 17n - 10$$

Certain binomial products occur so frequently that their patterns should be memorized.

Special Products

Square of a Sum: $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

Square of a Difference: $(a-b)^2 = (a-b)(a-b) = a^2 - 2ab + b^2$

Product of a Sum and Difference: $(a+b)(a-b) = a^2 - b^2$

Example 2 □ Simplify.

a. $(2a+3b)^2$ b. $(5n-4)(5n+4)$

c. $(x - \frac{1}{2}y)^2$

Solution □ a. $(2a+3b)^2 = (2a)^2 + 2(2a)(3b) + (3b)^2$ $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

$$= 2a^2 + 12ab + 9b^2$$

b. $(5n-4)(5n+4) = (5n)^2 - (4)^2$ $(a+b)(a-b) = a^2 - b^2$

$$= 25n^2 - 16$$

c. $(x - \frac{1}{2}y)^2 = (x)^2 - 2(x)(\frac{1}{2}y) + (\frac{1}{2}y)^2$ $(a-b)^2 = (a-b)(a-b) = a^2 - 2ab + b^2$

$$= x^2 - xy + \frac{1}{4}y^2$$

Exercises – FOIL Method and Special Products

1

$$(x+3)(x-5)$$

Which of the following is equivalent to the expression shown above?

A) $(x+1)^2-14$

B) $(x-1)^2-12$

C) $(x-1)^2-16$

D) $(x-2)^2-12$

2

$$(2-5x)(5x+2)$$

Which of the following is equivalent to the expression shown above?

A) $25x^2-4$

B) $-25x^2+4$

C) $25x^2-10x+4$

D) $-25x^2+10x+4$

3

$$4x^2-12xy+9y^2$$

Which of the following is equivalent to the expression shown above?

A) $(2x^2-3y)^2$

B) $(2x^2-3y^2)^2$

C) $(2x-3y^2)^2$

D) $(2x-3y)^2$

4

$$(x+y)(x-y)(x^2+y^2)$$

Which of the following is equivalent to the expression shown above?

A) $x^4-2x^2y^2+y^4$

B) $x^4+2x^2y^2+y^4$

C) x^4+y^4

D) x^4-y^4

5

What is the value of $\dfrac{3^{(a-b)}\cdot 3^{(a+b)}}{3^{2a+1}}$?

A) $\dfrac{1}{3}$

B) $\dfrac{1}{9}$

C) 3

D) 9

6

What is the value of $\dfrac{2^{(a-1)(a+1)}}{2^{(a-2)(a+2)}}$?

A) $\dfrac{1}{16}$

B) $\dfrac{1}{8}$

C) 8

D) 16

10-4. Prime Factorization, GCF, and LCM

A **prime number** is an integer greater than 1 that only has the factors 1 and itself.
The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19 23, and 29.

A **composite number** is a positive integer greater than 1 that has factors other than 1 and itself.
The numbers 0 and 1 are neither prime nor composite.

A whole number expressed as the product of prime factors is called the **prime factorization** of the number.

Example 1 □ Find the prime factorization of 420.

Solution □ Method 1 Find the least prime factors. Continue until all factor are prime.

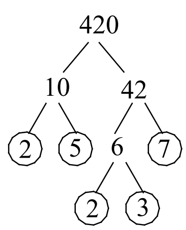
$420 = 2 \cdot 210$ The least prime factor of 420 is 2.

$= 2 \cdot 2 \cdot 105$ The least prime factor of 210 is 2.

$= 2 \cdot 2 \cdot 3 \cdot 35$ The least prime factor of 105 is 3.

$= 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$ The least prime factor of 35 is 5.

Method 2 Use a factor tree.



$420 = 10 \cdot 42$

$10 = 2 \cdot 5, \quad 42 = 6 \cdot 7$

$6 = 2 \cdot 3$

All of the factors in the last step are prime. Thus the prime factorization of 420 is $2^2 \cdot 3 \cdot 5 \cdot 7$.

A monomial is in **factored form** when it is expressed as the product of prime numbers and variables, and no variables has an exponent greater than 1.

Example 2 □ Factor completely.

a. $102a^2b^4$ b. $-28p^2q^3r$

Solution □ a. $102a^2b^4 = 2 \cdot 51 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b$ $102 = 2 \cdot 51$, $a^2 = a \cdot a$, $b^4 = b \cdot b \cdot b \cdot b$

$= 2 \cdot 3 \cdot 17 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b$ $51 = 3 \cdot 17$

Thus, $102a^2b^4$ in factored form is $2 \cdot 3 \cdot 17 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b$.

b. $-28p^2q^3r = -1 \cdot 28p^2q^3r$ Express -28 as $-1 \cdot 28$.

$= -1 \cdot 2 \cdot 14 \cdot p \cdot p \cdot q \cdot q \cdot q \cdot r$ $28 = 2 \cdot 14$

$= -1 \cdot 2 \cdot 2 \cdot 7 \cdot p \cdot p \cdot q \cdot q \cdot q \cdot r$ $14 = 2 \cdot 7$

Thus, $-28p^2q^3r$ in factored form is $-1 \cdot 2 \cdot 2 \cdot 7 \cdot p \cdot p \cdot q \cdot q \cdot q \cdot r$.

The **greatest common factor** (**GCF**) of two or more integers is the greatest integer that is a factor of each integer. To find the GCF of two or more monomials, take the smallest power of each prime factor and multiply.

The **least common multiple** (**LCM**) of two or more integers is the least positive integer that is a common multiple of two or more integers.
To find the LCM of two or more monomials, take the largest power of each prime factor and multiply.

Example 3 □ Find the GCF and LCM of each set of polynomials.

a. 90, 108

b. $12a^2b^3c$, $36a^5b^2c^2$

c. $28p^3q^4r^5$, $35p^2q^7r^4$, $42p^3q^6r^9$

d. $(x-y)^2(x+y)$, $(x-y)(x+y)^3$

Solution □ a. $90 = 10 \cdot 9 = 2 \cdot 5 \cdot 3 \cdot 3 = 2 \cdot 3^2 \cdot 5$

$108 = 2 \cdot 54 = 2 \cdot 2 \cdot 27 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 2^2 \cdot 3^3$

The smallest power of 2 is 2. The greatest power of 2 is 2^2.
The smallest power of 3 is 3^2. The greatest power of 3 is 3^3.
5 is not a common factor. The greatest power of 5 is 5.
The GCF is $2 \cdot 3^2$ or 18. The LCM is $2^2 \cdot 3^3 \cdot 5$ or 540.

b. $12a^2b^3c = 2^2 \cdot 3a^2b^3c$

$36a^5b^2c^2 = 2^2 \cdot 3^2 a^5b^2c^2$

The smallest power of 2 is 2^2. The greatest power of 2 is 2^2.
The smallest power of 3 is 3. The greatest power of 3 is 3^2.
The smallest power of a is a^2. The greatest power of a is a^5.
The smallest power of b is b^2. The greatest power of b is b^3.
The smallest power of c is c. The greatest power of c is c^2.
The GCF is $2^2 \cdot 3a^2b^2c$ or $12a^2b^2c$. The LCM is $2^2 \cdot 3^2 \cdot a^5b^3c^2$ or $36a^5b^3c^2$.

c. $28p^3q^4r^5 = 2^2 \cdot 7p^3q^4r^5$

$35p^2q^7r^4 = 5 \cdot 7p^2q^7r^4$

$42p^3q^6r^9 = 2 \cdot 3 \cdot 7p^3q^6r^9$

2 is not a common factor. The greatest power of 2 is 2^2.
3 is not a common factor. The greatest power of 3 is 3.
5 is not a common factor. The greatest power of 5 is 5.
The smallest power of 7 is 7. The greatest power of 7 is 7.
The smallest power of p is p^2. The greatest power of p is p^3.
The smallest power of q is q^4. The greatest power of q is q^7.
The smallest power of r is r^4. The greatest power of r is r^9.
The GCF is $7p^2q^4r^4$. The LCM is $2^2 \cdot 3 \cdot 5 \cdot 7p^3q^7r^9$ or $420p^3q^7r^9$.

d. $(x-y)^2(x+y)$

$(x-y)(x+y)^3$

The smallest power of $(x-y)$ is $(x-y)$ and the greatest power of $(x-y)$ is $(x-y)^2$.
The smallest power of $(x+y)$ is $(x+y)$ and the greatest power of $(x+y)$ is $(x+y)^3$
The GCF is $(x-y)(x+y)$ and the LCM is $(x-y)^2(x+y)^3$.

Exercises – Prime Factorization, GCF, and LCM

1

$$42x^2y^2 + 63xy^3$$

Which of the following is equivalent to the expression shown above?

A) $21x^2y^2(2x+3y)$

B) $21xy^2(2x+3y)$

C) $21x^2y(2x+3y)$

D) $21xy(2x+3y)$

2

$$12x^2y - 18xy^2z$$

Which of the following is equivalent to the expression shown above?

A) $6xy(2x-3yz)$

B) $6x^2y(2x-3yz)$

C) $6xy^2(2x-3yz)$

D) $6x^2y^2(2x-3yz)$

3

$$5a^2b - 10abc + 5bc^2$$

Which of the following is equivalent to the expression shown above?

A) $5b(a-b)^2$

B) $5c(a-b)^2$

C) $5a(b-c)^2$

D) $5b(a-c)^2$

4

If x and y are positive integers and $12^3 = 2^x \cdot 3^y$, what is the value of $x+y$?

5

If $2\times5^9 - k\times5^8 = 2\times5^8$, what is the value of k?

6

If $12^{99} - 12^{97} = 12^{97}\times n$, what is the value of n?

10-5. Factoring Using the Distributive Property

You used the Distributive Property to multiply a polynomial by a monomial. You can reverse this process to express a polynomial as the product of a monomial factor and a polynomial factor.

The first step in **factoring** a polynomial is to find the GCF of its terms. Then write each term as the product of the GCF and its remaining factors, and use the Distributive Property to factor out the GCF.

Factoring by Grouping.
To factor polynomials having four or more terms, group pairs of terms with common factors and use Distributive Property.

$$\begin{aligned} ax+bx+ay+by &= x(a+b)+y(a+b) \\ &= (a+b)(x+y) \end{aligned}$$

Example 1 □ Factor each polynomial.

a. $12x^3-18x$ b. $6mn-3n+2mp-p$

Solution □ a. $12x^3-18x$

$=6x(2x^2)-6x(3)$ Rewrite each term using the GCF.

$=6x(2x^2-3)$ Use the distributive property.

b. $6mn-3n+2mp-p$

$=(6mn-3n)+(2mp-p)$ Group terms with common factors.

$=3n(2m-1)+p(2m-1)$ Factor the GCF from each grouping.

$=(2m-1)(3n+p)$ Use the distributive property.

Another helpful tool in factoring polynomials is recognizing factors that are opposites of each other.

Factors	Opposit Factors
$a-b$	$-(a-b)=-a+b=b-a$
$9-x^2$	$-(9-x^2)=-9+x^2=x^2-9$
$p+2q-r$	$-(p+2q-r)=-p-2q+r$

Example 2 □ Factor each polynomial.

a. $3(x-y)-2x(y-x)$ b. $(a-2b-3)-(6c+4bc-2ac)$

Solution □ a. $3(x-y)-2x(y-x)$

$=3(x-y)-2x(-(x-y))$ $y-x=-(x-y)$

$=3(x-y)+2x(x-y)$ $-(-(x-y))=x-y$

$=(x-y)(3+2x)$ Use the distributive property.

b. $(a-2b-3)-(6c+4bc-2ac)$

$=(a-2b-3)-2c(3+2b-a)$ Factor the GCF.

$=(a-2b-3)+2c(-(3+2b-a))$ $-2c(3+2b-a)=2c(-(3+2b-a))$

$=(a-2b-3)+2c(a-2b-3)$ $-(3+2b-a)=a-2b-3$

$=(a-2b-3)(1+2c)$ Use the distributive property.

Exercises – Factoring Using the Distributive Property

1

$$1+2x-x(1+2x)$$

Which of the following is equivalent to the expression shown above?

A) $(1-2x)^2$

B) $(1+2x)(1-x)$

C) $-x(1+2x)$

D) $x(1-2x)$

2

What is the value of x, if $rx+sx=3$ and $r+s=\frac{1}{3}$?

A) 1

B) 3

C) 9

D) 27

3

$$2ax-6a-3x+9$$

Which of the following is equivalent to the expression shown above?

A) $(2a-1)(x-9)$

B) $(2a-3)(2x-3)$

C) $(a-3)(2x-3)$

D) $(2a-3)(x-3)$

4

$$mn-5n-m+5$$

Which of the following is equivalent to the expression shown above?

A) $(m-5)(n-1)$

B) $(m-1)(n-5)$

C) $(m+5)(n+1)$

D) $(m-5)(5n-1)$

5

$$7y^2-21xy-2y+6x$$

Which of the following is equivalent to the expression shown above?

A) $(7y-3)(y-2x)$

B) $(7y-2)(2y-3x)$

C) $(7y-2)(y-3x)$

D) $(7y+2)(2y-3x)$

6

$$x-2y+3z-2wx+4wy-6wz$$

Which of the following is equivalent to the expression shown above?

A) $(1+2w)(x+2y-3z)$

B) $(1-2w)(x-2y+3z)$

C) $(1+2w)(x-2y-3z)$

D) $(1-2w)(x-y-3z)$

Chapter 10 Practice Test

1

$$\frac{2^{(a+b)^2}}{2^{(a-b)^2}}$$

Which of the following is equivalent to the expression shown above?

A) $8^{(a+b)}$

B) 8^{ab}

C) 16^{a+b}

D) 16^{ab}

2

$$2m^2n - mnp - 6m + 3p$$

Which of the following is equivalent to the expression shown above?

A) $(2m-n)(mp-3)$

B) $(2m-p)(mn-3)$

C) $(2m+p)(mn+3)$

D) $(2m-n)(mn-3p)$

3

$$(\frac{a+b}{2})^2 - (\frac{a-b}{2})^2 =$$

A) ab

B) $-ab$

C) $\frac{2ab+b^2}{2}$

D) $ab+b^2$

4

If $(x+\frac{1}{x})^2 = 9$, then $(x-\frac{1}{x})^2 =$

A) 3

B) 5

C) 7

D) 9

5

If $8^{\frac{4}{3}} \cdot 8^{-\frac{8}{3}} = \frac{1}{2^m}$, what is the value of m?

A) $-\frac{4}{3}$

B) -4

C) $\frac{4}{3}$

D) 4

6

If $xy \neq 0$, then $\frac{(-2xy^2)^3}{4x^4y^5} =$

A) $-\frac{xy}{2}$

B) $-\frac{2}{x}$

C) $-\frac{2y}{x^2}$

D) $-\frac{2y}{x}$

7

If $x^{12} = 32n^4$ and $x^9 = 4n$, then $x =$

A) $2n$

B) $2n^{\frac{1}{2}}$

C) $4n^{\frac{1}{2}}$

D) $4n$

8

$$(3x^3 - 2x^2 - 7) - (-2x^2 + 6x + 2)$$

Which of the following is equivalent to the expression shown above?

A) $3(x^3 + 2x - 6)$

B) $3(x^3 - 2x - 9)$

C) $3(x^3 + 2x - 3)$

D) $3(x^3 - 2x - 3)$

9

$$9x - (x - 3)(x + 12)$$

Which of the following is equivalent to the expression shown above?

A) $36 - 18x - x^2$

B) $36 + 12x - x^2$

C) $(6 - x)(6 + x)$

D) $(6 - x)^2$

10

If $\dfrac{(2.1\times10^{-3})(2\times10^{5})}{7\times10^{-4}} = 6\times10^{n}$, what is the value of n?

11

If $a^{\frac{3}{4}} = 8$, what is the value of $a^{-\frac{1}{2}}$?

12

$$\frac{x^2 - x - a}{x - 2} = x + 1 - \frac{8}{x - 2}$$

In the equation above, what is the value of a?

Answer Key

Section 10-1

1. 9	2. $\frac{3}{16}$	3. 9	4. 155	5. D
6. 3	7. $\frac{2}{3}$	8. 9	9. 4	

Section 10-2

1. A	2. C	3. B	4. B	5. 2
6. 3				

Section 10-3

1. C	2. B	3. D	4. D	5. A
6. C				

Section 10-4

1. B	2. A	3. D	4. 9	5. 8
6. 143				

Section 10-5

1. B	2. C	3. D	4. A	5. C
6. B				

Chapter 10 Practice Test

1. B	2. B	3. A	4. B	5. D
6. D	7. A	8. D	9. C	10. 5
11. $\frac{1}{4}$	12. 10			

Answers and Explanations

Section 10-1

1. 9

$$(-a^2b^3)(2ab^2)(-3b)$$
$$=(-1)(2)(-3)a^2ab^3b^2b$$
$$=6a^3b^6=ka^mb^n$$

If the equation is true, $m=3$ and $n=6$, thus $m+n=3+6=9$.

2. $\frac{3}{16}$

$$(\frac{2}{3}a^2b)^2(\frac{4}{3}ab)^{-3}$$
$$=\frac{(\frac{2}{3}a^2b)^2}{(\frac{4}{3}ab)^3} \qquad a^{-n}=\frac{1}{a^n}$$
$$=\frac{\frac{4}{9}a^4b^2}{\frac{64}{27}a^3b^3}$$
$$=\frac{4}{9}\cdot\frac{27}{64}\frac{a}{b}=\frac{3}{16}\frac{a}{b}$$

If $(\frac{2}{3}a^2b)^2(\frac{4}{3}ab)^{-3}=ka^mb^n$, then $k=\frac{3}{16}$.

3. 9

$$\frac{(x)^3(-y)^2z^{-2}}{(x)^{-2}y^3z}=\frac{x^3y^2(x)^2}{y^3zz^2} \qquad a^{-n}=\frac{1}{a^n} \text{ and } \frac{1}{a^{-n}}=a^n$$
$$=\frac{x^5y^2}{y^3z^3}=\frac{x^5}{yz^3}=\frac{x^m}{y^nz^p}$$

If the equation is true, $m=5$, $n=1$, and $p=3$, thus $m+n+p=5+1+3=9$.

4. 155

$$2^x+2^{2x}+2^{3x}$$
$$=2^x+(2^x)^2+(2^x)^3 \qquad (a^m)^n=a^{m\cdot n}$$
$$=(5)+(5)^2+(5)^3 \qquad 2^x=5$$
$$=155$$

5. D

$$(3^x+3^x+3^x)\cdot 3^x$$
$$=(3\cdot 3^x)\cdot 3^x$$
$$=(3^{1+x})\cdot 3^x \qquad a^ma^n=a^{m+n}$$
$$=3^{1+2x} \qquad a^ma^n=a^{m+n}$$

6. 3

$$\frac{(6xy^2)(2xy)^2}{8x^2y^2}$$
$$=\frac{(6xy^2)(4x^2y^2)}{8x^2y^2}$$
$$=\frac{24x^3y^4}{8x^2y^2}=3xy^2$$

If the expression above is written in the form ax^my^n, $a=3$, $m=1$, and $n=2$.
Therefore, $m+n=1+2=3$.

7. $\frac{2}{3}$

$$\frac{(2x)^3(3x)}{(6x^2)^2} = \frac{(8x^3)(3x)}{36x^4} = \frac{24\cancel{x^4}}{36\cancel{x^4}} = \frac{2}{3}$$

8. 9

$$8,200 \times 300,000 = 8.2 \times 10^3 \times 3 \times 10^5$$
$$= 24.6 \times 10^8 = 2.46 \times 10 \times 10^8 = 2.46 \times 10^9$$

9. 4

$$\frac{24\cancel{0}}{80,00\cancel{0}} \times \frac{6,\cancel{000}}{900,\cancel{000}}$$
$$= \frac{24 \times 6}{8,000 \times 900}$$
$$= \frac{144}{72 \times 10^5} = \frac{2}{10^5}$$
$$= \frac{2}{10 \times 10^4} = \frac{1}{5 \times 10^4}$$

If the above expression is equal to $\frac{1}{5 \times 10^n}$, then the value of n is 4.

Section 10-2

1. A

$$a(2-a)+(a^2+3)-(2a+1)$$
$$= 2a - a^2 + a^2 + 3 - 2a - 1$$
$$= 2$$

2. C

$$(-m^2n - n^2 + 3mn^2) - (m^2n - n^2 + mn^2)$$
$$= -m^2n - \cancel{n^2} + 3mn^2 - m^2n + \cancel{n^2} - mn^2$$
$$= -2m^2n + 2mn^2$$

3. B

$$(2x^2 - 3x + 1) - (-2x^2 - 3x + 2)$$
$$= 2x^2 - \cancel{3x} + 1 + 2x^2 + \cancel{3x} - 2$$
$$= 4x^2 - 1$$

If the expression above is written in the form $ax^2 + bx + c$, $a = 4$, $b = 0$, and $c = -1$. Therefore, $a + b + c = 4 + 0 + (-1) = 3$.

4. B

	$x^2 + 3$	Quotient
$x-1$	$\overline{)\,x^3 - x^2 + 3x - 3}$	Dividend
	$x^3 - x^2$	$x^2 \times (x-1) = x^3 - x^2$
	0	Result of subtraction
	$3x - 3$	
	$3x - 3$	$3 \times (x-1) = 3x - 3$
	0	Result of subtraction

Therefore, $(x^3 - x^2 + 3x - 3) \div (x - 1) = x^2 + 3$.

5. 2

$$(14x^2 + 9x - 20) \div (ax - 1) = 7x + 8 + \frac{-12}{ax - 1}$$

Multiply each side of the equation by $ax - 1$.

$$(ax-1)[14x^2 + 9x - 20) \div (ax - 1)]$$
$$= (ax-1)[7x + 8 + \frac{-12}{ax-1}]$$
$$\Rightarrow\ 14x^2 + 9x - 20 = (ax-1)(7x+8) + (-12)$$
$$\Rightarrow\ 14x^2 + 9x - 20 = 7ax^2 + (8a - 7)x - 20$$

The coefficients of x- terms have to be equal, so $9 = 8a - 7$.

$$14x^2 + 9x - 20 = 7ax^2 + (8a-7)x - 20$$

The coefficients of x^2- terms have to be equal, so $14 = -7a$.

Since the coefficients of x^2- terms have to be equal on both sides of the equation, $14 = 7a$, or $a = 2$.

6. 3

$$\frac{6x^2 - 5x + 4}{-3x + 1} = -2x + 1 + \frac{A}{-3x + 1}$$

Multiply each side of the equation by $-3x + 1$.

$$(-3x+1)[\frac{6x^2 - 5x + 4}{-3x + 1}]$$
$$= (-3x+1)[-2x + 1 + \frac{A}{-3x+1}]$$
$$\Rightarrow\ 6x^2 - 5x + 4 = 6x^2 - 5x + 1 + A$$

Since the constant terms have to be equal on both sides of the equation, $4 = 1 + A$, or $A = 3$

Section 10-3

1. C

$(x+3)(x-5)=x^2-5x+3x-15$

$=x^2-2x-15$

Choice A gives x-term $+2$ and constant term -13.
Choice B gives x-term -2 and constant term -11.
Choice C gives x-term -2 and constant term -15.
Choice C is correct.

2. B

$(2-5x)(5x+2)$

$=(2)(5x)+(2)(2)-(5x)(5x)-(5x)(2)$

$=10x+4-25x^2-10x$

$=4-25x^2$

3. D

$4x^2-12xy+9y^2$

$=(2x)^2-2(2x)(3y)+(3y)^2$

$=(2x-3y)^2$

4. D

$(x+y)(x-y)(x^2+y^2)$

$=(x^2-y^2)(x^2+y^2)$ $\quad (x+y)(x-y)=x^2-y^2$

$=x^2x^2+\cancel{x^2y^2}-\cancel{y^2x^2}-y^2y^2$

$=x^4-y^4$

5. A

$\dfrac{3^{(a-b)}\cdot 3^{(a+b)}}{3^{2a+1}}$

$=3^{(a-b)+(a+b)-(2a+1)}$ $\quad a^m a^n=a^{m+n}$ and $\dfrac{a^m}{a^n}=a^{m-n}$

$=3^{-1}=\dfrac{1}{3}$

6. C

$\dfrac{2^{(a-1)(a+1)}}{2^{(a-2)(a+2)}}$

$=\dfrac{2^{(a^2-1)}}{2^{(a^2-4)}}$ $\quad$ FOIL

$=2^{(a^2-1)-(a^2-4)}$ $\quad \dfrac{a^m}{a^n}=a^{m-n}$

$=2^3=8$

Section 10-4

1. B

$42x^2y^2+63xy^3$

$=21xy^2(2x+3y)$ $\quad$ GCF is $21xy^2$.

2. A

$12x^2y-18xy^2z$

$=6xy(2x-3yz)$ $\quad$ GCF is $6xy$.

3. D

$5a^2b-10abc+5bc^2$

$=5b(a^2-2ac+c^2)$ $\quad$ GCF is $5b$.

$=5b(a-c)^2$ $\quad (a-c)^2=a^2-2ac+c^2$

4. 9

$12^3=2^x\cdot 3^y$

$(2^2\cdot 3)^3=2^x\cdot 3^y$ $\quad 12=2^2\cdot 3$

$2^6\cdot 3^3=2^x\cdot 3^y$ $\quad (2^2)^3=2^6$

So, we can conclude that $x=6$ and $y=3$.
Therefore, $x+y=6+3=9$.

5. 8

$2\times 5^9-k\times 5^8=2\times 5^8$

$2\times 5\cdot 5^8-k\times 5^8=2\times 5^8$ $\quad 5^9=5\cdot 5^8$

$10\cdot 5^8-k\times 5^8=2\times 5^8$ $\quad$ Simplify.

$(10-k)5^8=2\times 5^8$ $\quad$ Factor.

Therefore, $10-k=2$, or $k=8$.

6. 143

$12^{99}-12^{97}=12^{97}\times n$

$12^2\times 12^{97}-12^{97}=12^{97}\times n$ $\quad 12^{99}=12^2\times 12^{97}$

$12^{97}(12^2-1)=12^{97}\times n$ $\quad$ Factor.

Therefore, $12^2-1=n$, or $n=143$.

Section 10-5

1. B

$1+2x-x(1+2x)$

$=1(1+2x)-x(1+2x)$

$=(1+2x)(1-x)$ $\quad$ GCF is $1+2x$.

2. C

$rx + sx = 3$

$x(r+s) = 3$ Factor.

$x(\frac{1}{3}) = 3$ Substitute $\frac{1}{3}$ for $r+s$.

$x = 9$

3. D

$2ax - 6a - 3x + 9$

$= (2ax - 6a) - (3x - 9)$ Group terms with common factors. $-3x + 9 = -(3x - 9)$

$= 2a(x-3) - 3(x-3)$ Factor the GCF.

$= (x-3)(2a-3)$ Distributive Property

4. A

$mn - 5n - m + 5$

$= (mn - 5n) - (m - 5)$ Group terms with common factors. $-m + 5 = -(m-5)$

$= n(m-5) - (m-5)$ Factor the GCF.

$= (m-5)(n-1)$ Distributive Property

5. C

$7y^2 - 21xy - 2y + 6x$

$= (7y^2 - 21xy) - (2y - 6x)$

$= 7y(y - 3x) - 2(y - 3x)$

$= (7y - 2)(y - 3x)$

6. B

$x - 2y + 3z - 2wx + 4wy - 6wz$

$= (x - 2y + 3z) - (2wx - 4wy + 6wz)$

$= (x - 2y + 3z) - 2w(x - 2y + 3z)$

$= (1 - 2w)(x - 2y + 3z)$

Chapter 10 Practice Test

1. B

$\frac{2^{(a+b)^2}}{2^{(a-b)^2}}$

$= 2^{(a+b)^2 - (a-b)^2}$ $\frac{a^m}{a^n} = a^{m-n}$

$= 2^{(a^2+2ab+b^2)-(a^2-2ab+b^2)}$

$= 2^{4ab}$

$= (2^4)^{ab}$ $(a^m)^n = a^{m \cdot n}$

$= (16)^{ab}$

2. B

$2m^2n - mnp - 6m + 3p$

$= (2m^2n - mnp) - (6m - 3p)$

$= mn(2m - p) - 3(2m - p)$

$= (2m - p)(mn - 3)$

3. A

$(\frac{a+b}{2})^2 - (\frac{a-b}{2})^2 = \frac{(a+b)^2}{4} - \frac{(a-b)^2}{4}$

$= \frac{a^2 + 2ab + b^2}{4} - \frac{a^2 - 2ab + b^2}{4}$

$= \frac{4ab}{4} = ab$

4. B

$(x + \frac{1}{x})^2 = 9$

$x^2 + 2x \cdot \frac{1}{x} + (\frac{1}{x})^2 = 9$

$x^2 + 2 + \frac{1}{x^2} = 9$

$x^2 + \frac{1}{x^2} = 7$

$(x - \frac{1}{x})^2 = x^2 - 2x \cdot \frac{1}{x} + \frac{1}{x^2}$

$= x^2 - 2 + \frac{1}{x^2} = x^2 + \frac{1}{x^2} - 2$

$= 7 - 2 = 5$ Substitute 7 for $x^2 + \frac{1}{x^2} = 7$.

5. D

$8^{\frac{4}{3}} \cdot 8^{-\frac{8}{3}} = 8^{\frac{4}{3} - \frac{8}{3}} = 8^{-\frac{4}{3}} = (2^3)^{-\frac{4}{3}}$

$= 2^{-4} = \frac{1}{2^4}$

If $8^{\frac{4}{3}} \cdot 8^{-\frac{8}{3}} = \frac{1}{2^m}$, then $m = 4$.

6. D

$\frac{(-2xy^2)^3}{4x^4y^5} = \frac{-8x^3y^6}{4x^4y^5}$

$= -\frac{2y}{x}$

7. A

Given $x^{12} = 32n^4$ and $x^9 = 4n$.

$x^{12} = 32n^4$

$\frac{x^{12}}{x^9} = \frac{32n^4}{x^9}$ Divide each side by x^9.

$x^3 = \frac{32n^4}{x^9}$ Simplify.

$x^3 = \frac{32n^4}{4n}$ Substitute $4n$ for x^9.

$x^3 = 8n^3$ Simplify.

$(x)^3 = (2n)^3$ $8n^3 = (2n)^3$

Therefore, $x = 2n$.

8. D

$(3x^3 - 2x^2 - 7) - (-2x^2 + 6x + 2)$

$= 3x^3 - 2x^2 - 7 + 2x^2 - 6x - 2$

$= 3x^3 - 6x - 9$

$= 3(x^3 - 2x - 3)$

9. C

$9x - (x-3)(x+12)$

$= 9x - (x^2 + 9x - 36)$

$= 9x - x^2 - 9x + 36$

$= 36 - x^2$

$= (6-x)(6+x)$

10. 5

$\frac{(2.1\times10^{-3})(2\times10^5)}{7\times10^{-4}}$

$= \frac{4.2\times10^2}{7\times10^{-4}}$

$= \frac{4.2\times10^2\times10^4}{7}$ $\frac{1}{a^{-n}} = a^n$

$= 0.6\times10^2\times10^4$

$= 0.6\times10^6$

$= 6\times10^5$

If $\frac{(2.1\times10^{-3})(2\times10^5)}{7\times10^{-4}} = 6\times10^n$, then $n = 5$.

11. $\frac{1}{4}$

$a^{\frac{3}{4}} = 8$

$(a^{\frac{3}{4}})^{\frac{4}{3}} = (8)^{\frac{4}{3}}$

$a = (2^3)^{\frac{4}{3}}$

$a = 2^4$

Therefore, $a^{-\frac{1}{2}} = (2^4)^{-\frac{1}{2}} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$.

12. 10

$\frac{x^2 - x - a}{x-2} = x + 1 - \frac{8}{x-2}$

Multiply each side of the equation by $x-2$.

$(x-2)[\frac{x^2-x-a}{x-2}] = (x-2)[x+1-\frac{8}{x-2}]$

$\Rightarrow \; x^2 - x - a = (x-2)(x+1) - 8$

$\Rightarrow \; x^2 - x - a = x^2 - x - 2 - 8$

$\Rightarrow \; x^2 - x - a = x^2 - x - 10$

Since the constant terms have to be equal on both sides of the equation, $a = 10$.